Risolvi le seguenti equazioni:

1. $$-\frac{2}{5}x-\frac{2}{5}x\left\{\frac{3}{2}x+\frac{1}{3}\left[x-\frac{2}{3}\left(\frac{2}{3}x-\frac{3}{2}\right)^2+\frac{2}{3}x\left(\frac{4}{9}x-\frac{7}{2}\right)\right]\right\}=\frac{1}{60}$$

$$-\frac{2}{5}x-\frac{2}{5}x\left\{\frac{3}{2}x+\frac{1}{3}\left[x-\frac{2}{3}\left(\frac{4}{9}x^2-2x+\frac{9}{4}\right)+\frac{8}{27}x^2-\frac{7}{3}x\right]\right\}=\frac{1}{60}$$

$$-\frac{2}{5}x-\frac{2}{5}x\left[\frac{3}{2}x+\frac{1}{3}\left(x-\frac{8}{27}x^2+\frac{4}{3}x-\frac{3}{2}+\frac{8}{27}x^2-\frac{7}{3}x\right)\right]=\frac{1}{60}$$

$$-2x-2x\left[\frac{3}{2}x+\frac{1}{3}\left(-\frac{3}{2}\right)\right]=\frac{1}{12}\qquad -2x-2x\left(\frac{3}{2}x-\frac{1}{2}\right)=\frac{1}{12}\qquad -2x-3x^2+x-\frac{1}{12}=0$$

$$3x^2+x+\frac{1}{12}=0\qquad 36x^2+12x+1=0\qquad (6x+1)^2=0\qquad x=-\frac{1}{6}$$

2. $$\frac{2(x-1)(x+1)+x\sqrt{2}}{x\sqrt{2}+2}=\frac{2x^2-2}{x\sqrt{2}}$$

$$\frac{2(x^2-1)+x\sqrt{2}}{\sqrt{2}(x+\sqrt{2})}=\frac{2x^2-2}{x\sqrt{2}}\qquad \frac{2x(x^2-1)+x^2\sqrt{2}-(2x^2-2)(x+\sqrt{2})}{x\sqrt{2}(x+\sqrt{2})}=0\qquad C.A.:\begin{cases}x\neq 0\\ x\neq -\sqrt{2}\end{cases}$$

$$2x^3-2x+x^2\sqrt{2}-2x^3-2x^2\sqrt{2}+2x+2\sqrt{2}=0$$

$$-x^2\sqrt{2}=-2\sqrt{2}\qquad x^2=2\qquad x_1=-\sqrt{2}\ non\ acc.\qquad x_2=\sqrt{2}\ acc.$$

3. $$\sqrt{3}\left(x+\frac{4}{x}\right)+2=2\left(1-\frac{\sqrt{3}}{x}\right)$$

$$x\sqrt{3}+\frac{4\sqrt{3}}{x}+2=2-\frac{2\sqrt{3}}{x}\qquad x\sqrt{3}+\frac{6\sqrt{3}}{x}=0\qquad x+\frac{6}{x}=0\qquad \frac{x^2+6}{x}=0\qquad \nexists x\in\mathbb{R}$$

4. $$\frac{x}{7x+\sqrt{7}}+\frac{1}{7}=\frac{x^2+7}{7x\sqrt{7}+7}$$

$$\frac{x}{\sqrt{7}(x\sqrt{7}+1)}+\frac{1}{(\sqrt{7})^2}=\frac{x^2+7}{7(x\sqrt{7}+1)}\qquad \frac{x\sqrt{7}+x\sqrt{7}+1-x^2-7}{7(x\sqrt{7}+1)}=0\qquad C.A.:\ x\neq-\frac{\sqrt{7}}{7}$$

$$x^2-2x\sqrt{7}+6=0\qquad x_{1,2}=\sqrt{7}\pm\sqrt{7-6}\qquad x_{1,2}=\sqrt{7}\pm 1\qquad acc.$$

5. $$\frac{(\sqrt{5}+1)^2}{x-2(\sqrt{5}+1)}+x=0$$

$$(\sqrt{5}+1)^2+x^2-2x(\sqrt{5}+1)=0\qquad C.A.:\ x\neq 2(1+\sqrt{5})$$

$$[(\sqrt{5}+1)-x]^2=0\qquad x_{1,2}=1+\sqrt{5}$$

Risolvi e discuti le seguenti equazioni letterali:

6. $4k^2x^2 - 4kx - 4k^2x + 1 + 2k = 0$

$4k^2x^2 - 2x(2k + 2k^2) + 1 + 2k = 0$

$Se\ \boldsymbol{k = 0}$ $1 = 0$ $\nexists \boldsymbol{x} \in \mathbb{R}$

$Se\ \boldsymbol{k \neq 0}$ $\frac{\Delta}{4} = (2k + 2k^2)^2 - 4k^2(1 + 2k) = 4k^2 + 8k^3 + 4k^4 - 4k^2 - 8k^3 = 4k^4$

$$x_{1,2} = \frac{2k + 2k^2 \pm 2k^2}{4k^2} = \begin{cases} \boldsymbol{\frac{1}{2k}} \\ \frac{2k(1+2k)}{4k^2} = \boldsymbol{\frac{1+2k}{2k}} \end{cases}$$

7. $k^2x^2 - 144\,k^3 = 0$

$k^2(x^2 - 144\,k) = 0$

$Se\ \boldsymbol{k = 0}$ $\forall \boldsymbol{x} \in \mathbb{R}$

$Se\ k \neq 0$ $x^2 = 144\,k$

$\boldsymbol{Se\ k > 0}$ $\boldsymbol{x_{1,2} = \pm 12\sqrt{k}}$ $\boldsymbol{Se\ k < 0}$ $\nexists \boldsymbol{x} \in \mathbb{R}$

8. $kx(x - 3) - x(x - 3k) = 16k - 16$

$kx^2 - 3kx - x^2 + 3kx = 16k - 16$ $x^2(k - 1) = 16(k - 1)$

$Se\ \boldsymbol{k = 1}$ $\forall \boldsymbol{x} \in \mathbb{R}$

$Se\ \boldsymbol{k \neq 1}$ $x^2 = 16$ $x_{1,2} = \boldsymbol{\pm 4}$

Data l'equazione parametrica: $x^2 - 2(k - 2)\,x + k^2 - 4 = 0$, stabilisci quale valore deve assumere k perché:

le soluzioni siano reali e distinte: $\frac{\Delta}{4} > 0$: $(k - 2)^2 - k^2 + 4 = k^2 - 4k + 4 - k^2 + 4 > 0$ $\boldsymbol{k < 2}$

le soluzioni siano coincidenti: $\frac{\Delta}{4} = 0$: $\boldsymbol{k = 2}$

le soluzioni siano opposte: $S = 0 \Rightarrow -\frac{b}{a} = 0 \Rightarrow b = 0 \Rightarrow k - 2 = 0 \Rightarrow \boldsymbol{k = 2}$

una delle soluzioni sia nulla: $k^2 - 4 = 0 \Rightarrow \boldsymbol{k = \pm 2}$

la somma dei reciproci delle soluzioni sia uguale a 2: $\frac{1}{x_1} + \frac{1}{x_2} = 2 \Rightarrow \frac{x_1 + x_2}{x_1 x_2} = 2 \Rightarrow \frac{2(k-2)}{k^2-4} = 2 \Rightarrow \frac{1}{k+2} = 1 \Rightarrow \boldsymbol{k = -1}$