42. $(2^x - 8)(2^{2x} - 3 \cdot 2^{x+1} + 8) \geq 0$

Primo fattore: $2^x - 8 \geq 0 \quad \Rightarrow \quad x \geq 3$

Secondo fattore: pongo $2^x = t \quad \Rightarrow \quad t^2 - 6t + 8 \geq 0 \quad \Rightarrow$

$t \leq 2 \ \vee \ t \geq 4 \quad \Rightarrow \quad 2^x \leq 2 \ \vee \ 2^x \geq 4$

$x \leq 1 \ \vee \ x \geq 2$

$1 \leq x \leq 2 \ \vee \ x \geq 3$

43. $(5^{3x} - 5^{2x})\left(e^{\frac{1}{x}} - e^2\right) \leq 0$

$5^{3x} - 5^{2x} \geq 0 \quad \Rightarrow \quad 5^{2x}(5^x - 1) \geq 0 \quad \Rightarrow \quad 5^x \geq 1 \quad \Rightarrow \quad x \geq 0$

$e^{\frac{1}{x}} \geq e^2 \quad \Rightarrow \quad \frac{1}{x} \geq 2 \quad \Rightarrow \quad 0 < x \leq \frac{1}{2}$ $\qquad x \geq \frac{1}{2}$

44. $4^{\frac{2}{x}} - 4^{\frac{1}{x}} + 1 > 0$

Pongo: $4^{\frac{1}{x}} = t \quad \Rightarrow \quad t^2 - t + 1 > 0 \quad \forall t \in R \quad \Rightarrow \quad \forall\ x \neq 0$

45. $30\left(\frac{2}{3}\right)^{\frac{x}{2}} - 27\left(\frac{2}{3}\right)^x - 8 \leq 0$

Pongo: $\left(\frac{2}{3}\right)^{\frac{x}{2}} = t \quad \Rightarrow \quad 30t - 27t^2 - 8 \leq 0 \quad \Rightarrow \quad t \leq \frac{4}{9} \ \vee \ t \geq \frac{2}{3}$

$\left(\frac{2}{3}\right)^{\frac{x}{2}} \leq \left(\frac{2}{3}\right)^2 \ \vee \ \left(\frac{2}{3}\right)^{\frac{x}{2}} \geq \left(\frac{2}{3}\right)^1 \quad \Rightarrow \quad x \leq 2 \ \vee \ x \geq 4$

46. $\dfrac{5^{|x+2|} - 5}{e^x - \sqrt{e}} \leq 0$

$N \geq 0:\ 5^{|x+2|} \geq 5^1 \quad \Rightarrow \quad |x+2| \geq 1$

$x + 2 \leq -1 \ \vee \ x + 2 \geq 1 \quad \Rightarrow \quad x \leq -3 \ \vee \ x \geq -1$

$D > 0:\ e^x > e^{\frac{1}{2}} \quad \Rightarrow \quad x > \frac{1}{2}$

$x \leq -3 \ \vee \ -1 \leq x < \frac{1}{2}$

47. $\left[\left(\frac{2}{3}\right)^{x} - \sqrt[3]{\frac{2}{3}}\right]\left(3\cdot 3^{x} - \frac{1}{3}\right) \geq 0$

$\left(\frac{2}{3}\right)^{x} \geq \sqrt[3]{\frac{2}{3}} \quad \Rightarrow \quad x \leq \frac{1}{3}$

$3^{x+1} \geq 3^{-1} \quad \Rightarrow \quad x+1 \geq -1 \quad \Rightarrow \quad x \geq -2$ $\qquad -2 \leq x \leq \frac{1}{3}$

48. $2^{3x-1} + (2^{x-1})^{3} \geq 5\cdot 2^{x}$

Pongo: $2^{x} = t \quad \Rightarrow \quad \frac{1}{2}t^{3} + \frac{1}{8}t^{3} \geq 5t \quad \Rightarrow \quad 5t\left(\frac{1}{8}t^{2} - 1\right) \geq 0$

$t^{2} - 8 \geq 0 \quad \Rightarrow \quad t \leq -\sqrt{8} \ \vee \ t \geq \sqrt{8}$

$2^{x} \geq 2^{\frac{3}{2}} \quad \Rightarrow \quad x \geq \frac{3}{2}$

49. $\dfrac{3^{-x} - 81}{5^{\frac{x+2}{x}} - 25} \leq 0$

$N \geq 0:\ 3^{-x} \geq 3^{4} \quad \Rightarrow \quad -x \geq 4 \quad \Rightarrow \quad x \leq -4$

$D > 0:\ 5^{\frac{x+2}{x}} > 5^{2} \quad \Rightarrow \quad \frac{x+2}{x} > 2 \quad \Rightarrow \quad \frac{2-x}{x} > 0 \quad \Rightarrow \quad 0 < x < 2$

$x \leq -4 \ \vee \ 0 < x < 2$

50. $\sqrt{2^{x^{2}} - \frac{1}{3}} \geq \frac{2}{3}$

$x^{2} \geq 0 \quad \forall\ x \in R$, perciò il valore più piccolo che può essere assunto da $2^{x^{2}}$ è 1, ma se

$2^{x^{2}} = 1 \quad \Rightarrow \quad \sqrt{1 - \frac{1}{3}} \geq \frac{2}{3} \quad \Rightarrow \quad \left(\frac{2}{3}\right)^{\frac{1}{2}} \geq \frac{2}{3}$ è un'affermazione vera, perciò: $\forall\ x \in R$

51. $\begin{cases} 3^{1-x} + 3^{1+x} > 6 \\ \left(\frac{1}{9}\right)^x - 8\left(\frac{1}{3}\right)^x \geq 9 \end{cases}$

$3^{1-x} + 3^{1+x} > 6$ Pongo: $3^x = t \quad \Rightarrow \quad \frac{3}{t} + 3t > 6$

$t^2 - 2t + 1 > 0 \quad \forall\, t \neq 1 \quad \Rightarrow \quad 3^x \neq 1 \quad \Rightarrow \quad x \neq 0$

$\left(\frac{1}{9}\right)^x - 8\left(\frac{1}{3}\right)^x \geq 9$ Pongo: $\left(\frac{1}{3}\right)^x = t \quad \Rightarrow \quad t^2 - 8t - 9 \geq 0$

$t \leq -1 \;\vee\; t \geq 9 \quad \Rightarrow \quad \left(\frac{1}{3}\right)^x \geq 9 \quad \Rightarrow \quad x \leq -2$ $\qquad x \leq -2$

52. $3^{4x} - 3^{3x} - 7 \cdot 3^{2x} + 3^x + 6 < 0$

Pongo: $3^x = t \quad \Rightarrow \quad t^4 - t^3 - 7t^2 + t + 6 < 0$

Applicando Ruffini ottengo: $t = \pm 1;\; t = 3;\; t = -2$

$-2 < t < -1 \;\vee\; 1 < t < 3 \quad \Rightarrow \quad 1 < 3^x < 3 \quad \Rightarrow \quad 0 < x < 1$

53. $4^{2x+1} - \frac{7}{3} \cdot 9^x > 7 \cdot 3^{2x} + 16^{x-1}$

$4^{2x+1} - 4^{2x-2} > \frac{7}{3} \cdot 3^{2x} + 7 \cdot 3^{2x}$

$4^{2x}\left(4 - \frac{1}{16}\right) > 7 \cdot 3^{2x}\left(\frac{1}{3} + 1\right)$

$\left(\frac{4}{3}\right)^{2x} \frac{63}{16} \cdot \frac{3}{28} > 1 \quad \Rightarrow \quad \left(\frac{4}{3}\right)^{2x} > \left(\frac{4}{3}\right)^3 \quad \Rightarrow \quad x > \frac{3}{2}$

54. $\frac{3 \cdot 2^x}{2^x - 2} + \frac{4}{2^x + 2} + \frac{3 \cdot 4^x - 8}{4 - 4^x} < 0$

Pongo: $2^x = t \quad \Rightarrow \quad \frac{3t}{t-2} + \frac{4}{t+2} + \frac{3t^2 - 8}{4 - t^2} < 0$

$\frac{3t}{t-2} + \frac{4}{t+2} - \frac{3t^2 - 8}{(t-2)(t+2)} < 0$

$\frac{3t(t+2) + 4(t-2) - 3t^2 + 8}{(t-2)(t+2)} < 0 \quad \Rightarrow \quad \frac{3t^2 + 6t + 4t - 8 - 3t^2 + 8}{(t-2)(t+2)} < 0$

$\frac{10t}{(t-2)(t+2)} < 0 \quad \Rightarrow \quad -2 < t < 2 \quad \Rightarrow \quad 2^x < 2$ $\qquad x < 1$