ESERCIZI SVOLTI SUL CALCOLO DEI LIMITI

1. Calcola i seguenti limiti:

$$\lim_{x\to 4} \frac{\log_2 x + 1}{3\log_4 x} = \frac{\log_2 4 + 1}{3\log_4 4} = \frac{2+1}{3\cdot 1} = \frac{3}{3} = \boxed{1}$$

$$\lim_{x\to 3} \cos\left(\frac{\log_3 x - 1}{x+3}\right) = \cos\left(\frac{\log_3 3 - 1}{3+3}\right) = \cos\left(\frac{1-1}{6}\right) = \cos 0 = \boxed{0}$$

$$\lim_{x\to 1} \frac{2x-1}{\log x - 3} = \frac{2\cdot 1 - 1}{\log 1 - 3} = \frac{2-1}{0-3} = \frac{1}{-3} = \boxed{-\frac{1}{3}}$$

$$\lim_{x\to 2} \frac{\log(x^2+x-5)}{2^x - 1} = \frac{\log(4+2-5)}{2^2-1} = \frac{\log 1}{4-1} = \frac{0}{3} = \boxed{0}$$

$$\lim_{x\to 2} \sqrt{x + \log_2 x} = \sqrt{2+\log_2 2} = \sqrt{2+1} = \boxed{\sqrt{3}}$$

$$\lim_{x\to \pi} \frac{e^{\cos x} + sen\, x}{\sqrt{1+tg\, x}} = \frac{e^{\cos\pi} + sen\,\pi}{\sqrt{1+tg\,\pi}} = \frac{e^{-1}+0}{\sqrt{1+0}} = \frac{e^{-1}}{1} = \boxed{\frac{1}{e}}$$

$$\lim_{x\to 0} \log(\cos x) = \log(\cos 0) = \log 1 = \boxed{0}$$

$$\lim_{x\to 1} \frac{2x-2+2^x}{\sqrt{1+\log x}} = \frac{2\cdot 1 - 2 + 2^1}{\sqrt{1+\log 1}} = \frac{2-2+2}{\sqrt{1+0}} = \frac{2}{1} = \boxed{2}$$

$$\lim_{x\to -1} sen\,(3^{x+1}-1) = sen\,(3^{-1+1}-1) = sen\,(1-1) = sen\, 0 = \boxed{0}$$

$$\lim_{x\to 1} \log(1-\log x) = \log(1-\log 1) = \log(1-0) = \log 1 = \boxed{0}$$

$$\lim_{x\to 2} \frac{\sqrt{x^2+x+1}}{\cos(\pi x)} = \frac{\sqrt{2^2+2+1}}{\cos(2\pi)} = \frac{\sqrt{7}}{1} = \boxed{\sqrt{7}}$$

$$\lim_{x\to 3} \frac{\log_3 x + \log_3 \frac{3}{x}}{x-2} = \lim_{x\to 3} \frac{\log_3 x + \log_3 3 - \log_3 x}{x-2} = \lim_{x\to 3} \frac{\log_3 3}{x-2} = \frac{1}{3-2} = \boxed{1}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{sen\, x + \cos x}{2x} = \frac{sen\,\frac{\pi}{2} + \cos\frac{\pi}{2}}{2\cdot\frac{\pi}{2}} = \frac{1+0}{\pi} = \boxed{\frac{1}{\pi}}$$

$$\lim_{x\to 2} \frac{2x^2 - x + 1}{2^{2x} - 2^x + 2} = \frac{2\cdot 2^2 - 2 + 1}{2^{2\cdot 2} - 2^2 + 2} = \frac{8-2+1}{16-4+2} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to \pi} (sen\, x + 2\cos x - 1) = sen\,\pi + 2\cos\pi - 1 = 0 + 2\cdot(-1) - 1 = -2 - 1 = \boxed{-3}$$

2. Calcola i seguenti limiti, risolvendo le forme indeterminate corrispondenti:

Forma indeterminata del tipo $+\infty - \infty$

- $$\lim_{x \to +\infty} \left(\sqrt{x+7} - \sqrt{x-5}\right) = \lim_{x \to +\infty} \left(\sqrt{x+7} - \sqrt{x-5}\right) \cdot \frac{\sqrt{x+7} + \sqrt{x-5}}{\sqrt{x+7} + \sqrt{x-5}} =$$
$$= \lim_{x \to +\infty} \frac{x+7-x+5}{\sqrt{x+7} + \sqrt{x-5}} = \lim_{x \to +\infty} \frac{12}{\sqrt{x+7} + \sqrt{x-5}} = 0$$

- $$\lim_{x \to +\infty} \frac{x}{\sqrt{2x-1} - \sqrt{2x+2}} = \lim_{x \to +\infty} \frac{x}{\sqrt{2x-1} - \sqrt{2x+2}} \cdot \frac{\sqrt{2x-1} + \sqrt{2x+2}}{\sqrt{2x-1} + \sqrt{2x+2}} =$$
$$= \lim_{x \to +\infty} \frac{x\left(\sqrt{2x-1} + \sqrt{2x+2}\right)}{2x-1-2x-2} = \lim_{x \to +\infty} \frac{x\left(\sqrt{2x-1} + \sqrt{2x+2}\right)}{-3} = -\infty$$

- $$\lim_{x \to +\infty} \left(\sqrt{x^2+1} - \sqrt{x-4}\right) = \lim_{x \to +\infty} \left(\sqrt{x^2+1} - \sqrt{x-4}\right) \cdot \frac{\sqrt{x^2+1} + \sqrt{x-4}}{\sqrt{x^2+1} + \sqrt{x-4}} =$$
$$= \lim_{x \to +\infty} \frac{x^2+1-x+4}{\sqrt{x^2+1} + \sqrt{x-4}} = \lim_{x \to +\infty} \frac{x^2-x+5}{\sqrt{x^2+1} + \sqrt{x-4}} = +\infty$$

- $$\lim_{x \to -\infty} \left(x^4 - x^2 - 9\right) = \lim_{x \to -\infty} x^4 \left(1 - \frac{1}{x^2} - \frac{9}{x^4}\right) = +\infty$$

Forma indeterminata del tipo $\frac{\infty}{\infty}$

- $$\lim_{x \to -\infty} \frac{x^6 - 3x^4}{2x^2 - 2x + 1} = \lim_{x \to -\infty} \frac{x^6\left(1 - \frac{3}{x^2}\right)}{x^2\left(2 - \frac{2}{x} + \frac{1}{x^2}\right)} = +\infty$$ il grado del num. è maggiore del grado del den.

- $$\lim_{x \to +\infty} \frac{2x^5 - x^3 + x^4}{x^5 - 6x^2} = \lim_{x \to +\infty} \frac{x^5\left(2 - \frac{1}{x^2} + \frac{1}{x}\right)}{x^5\left(1 - \frac{6}{x^3}\right)} = 2$$ il grado del num. è uguale al grado del den.

- $$\lim_{x \to +\infty} \frac{x^2 - 2x + 3x^3}{2x^4 - x^2} = \lim_{x \to +\infty} \frac{x^3\left(\frac{1}{x} - \frac{2}{x^2} + 3\right)}{x^4\left(2 - \frac{1}{x^2}\right)} = 0$$ il grado del num. è minore del grado del den.

- $$\lim_{x \to +\infty} \frac{\sqrt{4x^2 - 3}}{x+1} = \lim_{x \to +\infty} \frac{x\sqrt{4 - \frac{3}{x^2}}}{x\left(1 + \frac{1}{x}\right)} = 2$$

- $$\lim_{x\to-\infty} \frac{3x-2}{\sqrt{x^2-x+1}} = \lim_{x\to-\infty} \frac{x\left(3-\frac{2}{x}\right)}{-x\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}} = \boxed{-3}$$

- $$\lim_{x\to+\infty} \frac{x^3+x+2}{\sqrt{2x^2+1}} = \lim_{x\to+\infty} \frac{x^3\left(1+\frac{1}{x^2}+\frac{2}{x^3}\right)}{x\sqrt{2+\frac{1}{x^2}}} = \boxed{+\infty}$$

Forma indeterminata del tipo $\frac{0}{0}$

- $$\lim_{x\to-5} \frac{x^2+3x-10}{x^2-25} = \lim_{x\to-5} \frac{(x+5)(x-2)}{(x+5)(x-5)} = \lim_{x\to-5} \frac{x-2}{x-5} = \boxed{\frac{7}{10}}$$

- $$\lim_{x\to-2} \frac{3x^2+x-10}{x^2-5x-14} = \lim_{x\to-2} \frac{(x+2)(3x-5)}{(x-7)(x+2)} = \lim_{x\to-2} \frac{3x-5}{x-7} = \boxed{\frac{11}{9}}$$

3. Determina gli asintoti delle seguenti funzioni, dopo averne determinato il dominio:

- $y = \dfrac{2x^2+3x}{x^2-1}$ $\qquad D_f =]-\infty; -1[\cup]-1; 1[\cup]1; +\infty[$

$\lim_{x\to-1^\pm} \dfrac{2x^2+3x}{(x-1)(x+1)} = \pm\infty$ $\qquad x = -1$ asintoto verticale

$\lim_{x\to1^\pm} \dfrac{2x^2+3x}{(x-1)(x+1)} = \pm\infty$ $\qquad x = 1$ asintoto verticale

$\lim_{x\to\pm\infty} \dfrac{2x^2+3x}{x^2-1} = 2$ $\qquad y = 2$ asintoto orizzontale

- $y = \dfrac{2x+5}{x^2-4x+4}$ $\qquad D_f =]-\infty; 2[\cup]2; +\infty[$

$\lim_{x\to2^\pm} \dfrac{2x+5}{(x-2)^2} = +\infty$ $\qquad x = 2$ asintoto verticale

$\lim_{x\to\pm\infty} \dfrac{2x+5}{x^2-4x+4} = 0$ $\qquad y = 0$ asintoto orizzontale

- $y = \dfrac{2x^2 + 3x}{x-1}$ $\quad D_f =]-\infty; 1[\cup]1; +\infty[$

$\lim\limits_{x \to 1^\pm} \dfrac{2x^2 + 3x}{x-1} = \pm\infty$ $\quad x = 1$ asintoto verticale

$\lim\limits_{x \to \pm\infty} \dfrac{2x^2 + 3x}{x-1} = \pm\infty$ $\quad$ può esistere asintoto obliquo

$m = \lim\limits_{x \to \pm\infty} \left(\dfrac{2x^2 + 3x}{x-1} \cdot \dfrac{1}{x} \right) = \lim\limits_{x \to \pm\infty} \dfrac{2x^2 + 3x}{x^2 - x} = 2$

$q = \lim\limits_{x \to \pm\infty} \left(\dfrac{2x^2 + 3x}{x-1} - 2x \right) = \lim\limits_{x \to \pm\infty} \dfrac{2x^2 + 3x - 2x^2 + 2x}{x-1} = 5$ $\quad y = 2x + 5$ as. obliquo

- $y = \dfrac{x^2 - 4x}{x^2 - 5x + 4}$ $\quad D_f =]-\infty; 1[\cup]1; 4[\cup]4; +\infty[$

$\lim\limits_{x \to 1^\pm} \dfrac{x^2 - 4x}{(x-1)(x-4)} = \pm\infty$ $\quad x = 1$ asintoto verticale

$\lim\limits_{x \to 4^\pm} \dfrac{x^2 - 4x}{(x-1)(x-4)} = \lim\limits_{x \to 4^\pm} \dfrac{x(x-4)}{(x-1)(x-4)} = \lim\limits_{x \to 4^\pm} \dfrac{x}{x-1} = \dfrac{4}{3}$

$\lim\limits_{x \to \pm\infty} \dfrac{x^2 - 4x}{x^2 - 5x + 4} = 1$ $\quad y = 1$ asintoto orizzontale

- $y = \dfrac{x^3 + 3x}{x-3}$ $\quad D_f =]-\infty; 3[\cup]3; +\infty[$

$\lim\limits_{x \to 3^\pm} \dfrac{x^3 + 3x}{x-3} = \pm\infty$ $\quad x = 3$ asintoto verticale

$\lim\limits_{x \to \pm\infty} \dfrac{x^3 + 3x}{x-3} = +\infty$ $\quad$ può esistere asintoto obliquo

$m = \lim\limits_{x \to \pm\infty} \left(\dfrac{x^3 + 3x}{x-3} \cdot \dfrac{1}{x} \right) = \lim\limits_{x \to \pm\infty} \dfrac{x^3 + 3x}{x^2 - 3x} = \pm\infty$ $\quad$ non esiste asintoto obliquo