1. Applicando la definizione di derivata calcola la derivata, in un punto generico, della funzione $y=\frac{x-2}{x+1}$.

$$\lim_{h\to 0}\frac{\frac{x+h-2}{x+h+1}-\frac{x-2}{x+1}}{h}=\lim_{h\to 0}\frac{x^2+hx-2x+x+h-2-x^2-xh-x+2x+2h+2}{h\,(x+1)(x+h+1)}=$$

$$=\lim_{h\to 0}\frac{3h}{h\,(x+1)(x+h+1)}=\lim_{h\to 0}\frac{3}{(x+1)(x+h+1)}=\frac{3}{(x+1)^2}$$

Calcola le derivate delle seguenti funzioni:

2. $y=7x^2+3x^5+\frac{1}{8}x^8-\frac{7}{18}x^{27}$

$$y'=14x+15x^4+x^7-\frac{21}{2}x^{26}$$

3. $y=\sqrt[7]{x^5}-\frac{1}{\sqrt[5]{x^4}}+x^2\sqrt{x}-\sqrt{\sqrt[5]{x^3}}$

$$y=x^{\frac{5}{7}}-x^{-\frac{4}{5}}+x^{\frac{5}{2}}-x^{\frac{3}{10}}\qquad y'=\frac{5}{7}x^{-\frac{2}{7}}+\frac{4}{5}x^{-\frac{9}{5}}+\frac{5}{2}x^{\frac{3}{2}}-\frac{3}{10}x^{-\frac{7}{10}}=\frac{5}{7\sqrt[7]{x^2}}+\frac{4}{5\sqrt[5]{x^9}}+\frac{5}{2}x\sqrt{x}-\frac{3}{10\sqrt[10]{x^7}}$$

4. $y=\frac{x^4-x\sqrt{x}}{\sqrt[4]{x}}$

$$y=x^4:x^{\frac{1}{4}}-x\cdot x^{\frac{1}{2}}:x^{\frac{1}{4}}=x^{\frac{15}{4}}-x^{\frac{5}{4}}\qquad y'=\frac{15}{4}x^{\frac{11}{4}}-\frac{5}{4}x^{\frac{1}{4}}=\frac{15}{4}x^3\sqrt[4]{x^3}-\frac{5}{4}\sqrt[4]{x}$$

5. $y=5^x-3^{x+2}$

$$y=5^x-9\cdot 3^x\qquad y'=5^x\ln 5-9\cdot 3^x\ln 3=5^x\ln 5-3^{x+2}\ln 3$$

6. $y=\log_4 x+x^5\log_3 7$

$$y'=\frac{1}{x}\log_4 e+5x^4\log_3 7$$

7. $y=\ln^2 x\cdot\log_x 8$

$$y=\ln^2 x\cdot\frac{\ln 8}{\ln x}=\ln 8\ln x\qquad y'=\frac{\ln 8}{x}$$

8. $y=\sin x-3\cos x$

$$y'=\cos x+3\sin x$$

9. $y=\sin 2x$

$$y=2\sin x\cos x\qquad y'=2\cos^2 x-2\sin^2 x=2\cos 2x$$