

$$1. \sqrt{\frac{1}{x} - \frac{1}{x^2}} + \sqrt{\frac{4x-4}{x^2}} = \sqrt{\frac{x-1}{x^2}} + 2\sqrt{\frac{x-1}{x^2}} = 3\sqrt{\frac{x-1}{x^2}} = \frac{3}{x}\sqrt{x-1}$$

C.E.: $\begin{cases} x-1 \geq 0 \\ x \neq 0 \end{cases} \quad x \geq 1$

$$2. \quad 2x\sqrt{\frac{x-2}{x^2}} - (x-1)\sqrt{\frac{x-2}{x^2-2x+1}}$$

C.E.: $\begin{cases} x-2 \geq 0 \\ x \neq 0 \\ x \neq 1 \end{cases} \quad x \geq 2$

$$= \frac{2x}{x}\sqrt{x-2} - (x-1)\sqrt{\frac{x-2}{(x-1)^2}} = 2\sqrt{x-2} - \frac{x-1}{x-1}\sqrt{x-2} = 2\sqrt{x-2} - \sqrt{x-2} = \sqrt{x-2}$$

$$3. \quad \sqrt{\frac{1}{x-1}\sqrt{x-1}} \cdot \sqrt[3]{\frac{x-1}{\sqrt{x-1}}} : \sqrt[12]{\frac{1}{(x-1)^7}}$$

$$= \sqrt{\sqrt{\frac{x-1}{(x-1)^2}}} \cdot \sqrt[3]{\sqrt{\frac{(x-1)^2}{x-1}}} \cdot \sqrt[12]{(x-1)^7} = \sqrt[4]{\frac{1}{x-1}} \cdot \sqrt[6]{x-1} \cdot \sqrt[12]{(x-1)^7} =$$

C.E.: $x > 1$

$$= \sqrt[12]{\frac{1}{(x-1)^3} \cdot (x-1)^2 \cdot (x-1)^7} = \sqrt[12]{(x-1)^6} = \sqrt{x-1}$$

$$4. \quad \sqrt{\frac{a^2-4}{a^2+4a+4}} \cdot \left(\sqrt[4]{\frac{a+2}{a-2}}\right)^2 \cdot \sqrt[4]{a^2-4a+4}$$

C.E.: $\begin{cases} (a-2)(a+2) \geq 0 \\ a^2+4a+4 \neq 0 \\ \frac{a+2}{a-2} \geq 0 \end{cases} \quad \begin{cases} a \leq -2 \vee a \geq 2 \\ a \neq -2 \\ a \leq -2 \vee a > 2 \end{cases} \quad a < -2 \vee a > 2$

$$= \sqrt{\frac{(a-2)(a+2)}{(a+2)^2}} \cdot \sqrt{\frac{a+2}{a-2}} \cdot \sqrt[4]{(a-2)^2} = \sqrt{\frac{a-2}{a+2}} \cdot \sqrt{\frac{a+2}{a-2}} \cdot \sqrt{|a-2|} = \sqrt{|a-2|}$$

$$5. \quad \sqrt{\frac{1}{x^2-1}\sqrt{\frac{x-1}{x+1}}} \cdot \sqrt{\frac{x+1}{4}} \cdot \sqrt[4]{16x^3-16x}$$

con $x > 1$

$$= \sqrt{\sqrt{\frac{1}{(x-1)^2(x+1)^2} \cdot \frac{x-1}{x+1}}} \cdot \frac{\sqrt{x+1}}{2} \cdot 2\sqrt[4]{x(x^2-1)} = \sqrt[4]{\frac{1}{(x-1)(x+1)^3}} \cdot \sqrt{x+1} \cdot \sqrt[4]{x(x-1)(x+1)} =$$

$$= \sqrt[4]{\frac{1}{(x+1)^2}} \cdot \sqrt{x+1} \cdot \sqrt[4]{x} = \sqrt{\frac{1}{x+1}} \cdot \sqrt{x+1} \cdot \sqrt[4]{x} = \sqrt[4]{x}$$

$$6. \quad \sqrt{\frac{x^2-5x+6}{x-1}} : \sqrt[3]{\frac{x-2}{1-x^2}} \cdot \sqrt[6]{\frac{x^2-1}{x-2}}$$

con $x \geq 3$

$$= \sqrt{\frac{(x-2)(x-3)}{x-1}} \cdot \sqrt[3]{-\frac{x^2-1}{x-2}} \cdot \sqrt[6]{\frac{(x-1)(x+1)}{x-2}} =$$

$$= -\sqrt[6]{\frac{(x-2)^3(x-3)^3}{(x-1)^3}} \cdot \frac{(x-1)^2(x+1)^2}{(x-2)^2} \cdot \frac{(x-1)(x+1)}{x-2} = -\sqrt[6]{(x-3)^3(x+1)^3} = -\sqrt{(x-3)(x+1)}$$

$$7. \quad \frac{1}{\sqrt{x+2}-\sqrt{x}} + \frac{1}{\sqrt{y+3}-\sqrt{y}} - \sqrt{\frac{x+2}{4}} - \sqrt{\frac{y+3}{9}} \quad \text{con } x \geq 0 \wedge y \geq 0$$

$$\begin{aligned} &= \frac{1}{\sqrt{x+2}-\sqrt{x}} \cdot \frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}} + \frac{1}{\sqrt{y+3}-\sqrt{y}} \cdot \frac{\sqrt{y+3}+\sqrt{y}}{\sqrt{y+3}+\sqrt{y}} - \frac{\sqrt{x+2}}{2} - \frac{\sqrt{y+3}}{3} = \\ &= \frac{\sqrt{x+2}+\sqrt{x}}{x+2-x} + \frac{\sqrt{y+3}+\sqrt{y}}{y+3-y} - \frac{\sqrt{x+2}}{2} - \frac{\sqrt{y+3}}{3} = \frac{\sqrt{x+2}}{2} + \frac{\sqrt{x}}{2} + \frac{\sqrt{y+3}}{3} + \frac{\sqrt{y}}{3} - \frac{\sqrt{x+2}}{2} - \frac{\sqrt{y+3}}{3} = \frac{\sqrt{x}}{2} + \frac{\sqrt{y}}{3} \end{aligned}$$