Calcola i seguenti integrali:

$$\int_1^6 \frac{x+4}{\sqrt{x+3}}\,dx$$

$$\int_1^6 \frac{x+3+1}{\sqrt{x+3}}\,dx = \int_1^6 \left(\sqrt{x+3}+\frac{1}{\sqrt{x+3}}\right)dx = \left[\frac{2}{3}\sqrt{(x+3)^3}+2\sqrt{x+3}\right]_1^6 =$$

$$= \frac{2}{3}\cdot 27 + 2\cdot 3 - \frac{2}{3}\cdot 8 - 2\cdot 4 = \boldsymbol{\frac{44}{3}}$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{4+\sin^2 x}\,dx$$

$$\frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\frac{\sin^2 x}{4}}\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\frac{1}{2}\cos x}{1+\left(\frac{\sin x}{2}\right)^2}\,dx = \left[\frac{1}{2}arc\tan\frac{\sin x}{2}\right]_0^{\frac{\pi}{2}} = \boldsymbol{\frac{1}{2}arc\tan\frac{1}{2}}$$

$$\int_{\frac{\pi}{2}}^{\frac{2}{3}\pi} \frac{1}{\sin x}\,dx$$

Risolvo l'integrale indefinito con le formule parametriche razionali:

$$\sin x = \frac{2t}{1+t^2} \qquad t = \tan\frac{x}{2} \qquad x = 2\,arc\tan t \qquad dx = \frac{2}{1+t^2}dt$$

$$\int \frac{1+t^2}{2t}\cdot\frac{2}{1+t^2}\,dt = \int \frac{1}{t}\,dt = \ln|t| + c$$

$$\int_{\frac{\pi}{2}}^{\frac{2}{3}\pi} \frac{1}{\sin x}dx = \left[\ln\left|\tan\frac{x}{2}\right|\right]_{\frac{\pi}{2}}^{\frac{2}{3}\pi} = \ln\sqrt{3} - \ln 1 = \boldsymbol{\ln\sqrt{3}}$$

$$\int_0^{\frac{1}{3}} 6x\,e^{3x}\,dx$$

Risolvo per parti:

$$f(x) = 2x \qquad f'(x) = 2 \qquad g'(x) = 3e^{3x} \qquad g(x) = e^{3x}$$

$$[2x\,e^{3x}]_0^{\frac{1}{3}} - 2\int_0^{\frac{1}{3}} e^{3x}\,dx = \frac{2}{3}e - 0 - \left[\frac{2}{3}e^{3x}\right]_0^{\frac{1}{3}} = \frac{2}{3}e - \frac{2}{3}e + \frac{2}{3} = \boldsymbol{\frac{2}{3}}$$

$$\int_0^1 arc\tan x\;dx$$

Risolvo per parti:

$$f(x) = arc\tan x \qquad f'(x) = \frac{1}{1+x^2} \qquad g'(x) = 1 \qquad g(x) = x$$

$$[x\,arc\tan x]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx = \frac{\pi}{4} - 0 - \left[\frac{1}{2}\ln(1+x^2)\right]_0^1 = \boldsymbol{\frac{\pi}{4} - \frac{1}{2}\ln 2}$$

$$\int_0^{\frac{1}{2}} \frac{x-4}{x^2+x-2}dx$$

$$\frac{x-4}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} = \frac{Ax-A+Bx+2B}{(x+2)(x-1)} = \frac{(A+B)x-A+2B}{(x+2)(x-1)}$$

$$\begin{cases} A+B=1 \\ -A+2B=-4 \end{cases} \qquad \begin{cases} A=2 \\ B=-1 \end{cases}$$

$$\int_0^{\frac{1}{2}} \left(\frac{2}{x+2} - \frac{1}{x-1}\right) dx = [2\ln|x+2| - \ln|x-1|]_0^{\frac{1}{2}} = 2\ln\frac{5}{2} - \ln\frac{1}{2} - 2\ln 2 - \ln 1 =$$

$$= \ln\left(\frac{25}{4} : \frac{1}{2} : 4\right) = \boldsymbol{ln\frac{25}{8}}$$

$$\int_0^2 \frac{x^3+6x+2}{x^2+4}dx$$

$$(x^3+6x+2) : (x^2+4) \qquad Q(x) = x \qquad R(x) = 2x+2$$

$$\int_0^2 \left(x + \frac{2x+2}{x^2+4}\right) dx = \int_0^2 \left(x + \frac{2x}{x^2+4} + \frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1}\right) dx =$$

$$= \left[\frac{1}{2}x^2 + \ln(x^2+4) + arc\tan\frac{x}{2}\right]_0^2 = 2 + \ln 8 + \frac{\pi}{4} - 0 - \ln 4 - 0 = \boldsymbol{2 + ln\,2 + \frac{\pi}{4}}$$