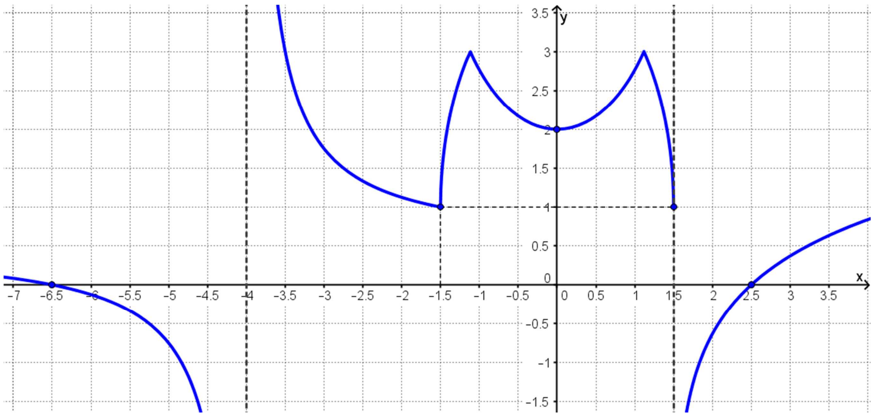
Dal grafico di $y = f(x)$ deduci le informazioni richieste: ________ / 4



Dominio:

$$\left]-\infty; -\mathbf{4}\right[\cup \left]-\mathbf{4}; \frac{\mathbf{3}}{\mathbf{2}}\right[\cup \left]\frac{\mathbf{3}}{\mathbf{2}}; +\infty\right[$$

Codominio: $\mathbb{R}$

Coordinate dei punti di intersezione con gli assi: $(\mathbf{-6,5; 0})$ $(\mathbf{2,5; 0})$ $(\mathbf{0; 2})$

Intervalli di positività:

$]-\infty; \mathbf{-6,5}[\ \cup\ ]\mathbf{-4; 1,5}]\ \cup\ ]\mathbf{2,5};\ +\infty[$

$\lim_{x\to-4^-} f(x) = -\infty$ $\lim_{x\to-4^+} f(x) = +\infty$ $\lim_{x\to\frac{3}{2}^-} f(x) = \mathbf{1}$ $\lim_{x\to\frac{3}{2}^+} f(x) = -\infty$ $\lim_{x\to\frac{3}{2}} f(x) = \nexists$

Calcola i seguenti limiti:

1. $\lim_{x\to 0} \frac{x^3 + 2x}{5x + 2\,sen\,x}$

$$\lim_{x\to 0} \frac{x^3 + 2x}{5x + 2\,sen\,x} = \lim_{x\to 0} \frac{x\,(x^2+2)}{x\left(5 + 2\frac{sen\,x}{x}\right)} = \frac{0+2}{5+2} = \frac{\mathbf{2}}{\mathbf{7}}$$

2. $\lim_{x\to -2^+} \frac{sen\,(x+2)}{x^2 + 4x + 4}$

$$\lim_{x\to -2^+} \frac{sen\,(x+2)}{(x+2)^2} = \lim_{x\to -2^+} \frac{sen\,(x+2)}{x+2} \cdot \frac{1}{x+2} = +\infty$$

3. $\lim_{x\to 2} \frac{2x^2 - x + 1}{2^{2x} - 2^x + 2}$

$$\lim_{x\to 2} \frac{2x^2 - x + 1}{2^{2x} - 2^x + 2} = \frac{8 - 2 + 1}{16 - 4 + 2} = \frac{\mathbf{1}}{\mathbf{2}}$$

4. $\lim_{x\to \pi} \frac{e^{\cos x} + sen\,x}{\sqrt{1 + tg\,x}}$

$$\lim_{x\to \pi} \frac{e^{\cos x} + sen\,x}{\sqrt{1 + tg\,x}} = \frac{e^{-1} + 0}{1} = \frac{\mathbf{1}}{\boldsymbol{e}}$$

5. $\lim_{x\to -\infty} \left(\frac{x+2}{x-1}\right)^x$ $\qquad cambio\ di\ variabile: \frac{x+2}{x-1} = 1 + \frac{1}{y} \qquad y = \frac{x-1}{3}$

$$\lim_{y\to -\infty} \left(1 + \frac{1}{y}\right)^{3y+1} = \lim_{y\to -\infty} \left[\left(1 + \frac{1}{y}\right)^y\right]^3 \cdot \left(1 + \frac{1}{y}\right) = e^3 \cdot 1 = \boldsymbol{e^3}$$

6. $\lim\limits_{x\to 0}\ (1+4x)^{\frac{5}{x}}$

$$\lim_{x\to 0}\ (1+4x)^{\frac{5}{x}} \underset{4x=\frac{1}{y}}{\overset{=}{\downarrow}} \lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{20y} = \lim_{y\to\infty}\left[\left(1+\frac{1}{y}\right)^{y}\right]^{20} = \boldsymbol{e^{20}}$$

7. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4}\right)$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4}\right)\cdot\frac{\sqrt{x^2+1}+\sqrt{x^2-4}}{\sqrt{x^2+1}+\sqrt{x^2-4}} = \lim_{x\to+\infty}\frac{x^2+1-x^2+4}{\sqrt{x^2+1}+\sqrt{x^2-4}} = \mathbf{0}$$

8. $\lim\limits_{x\to+\infty}\dfrac{x^2+3x^4}{2x^2-x+5x^4} = \lim\limits_{x\to+\infty}\dfrac{3x^4}{5x^4} = \mathbf{\dfrac{3}{5}}$

9. $\lim\limits_{x\to 0}\ 2x\ ctg\ x$

$$\lim_{x\to 0}\frac{2x\cos x}{senx} = 2\cdot\lim_{x\to 0}\frac{x}{senx}\cdot\cos x = 2\cdot\lim_{x\to 0}\left(\frac{sen\ x}{x}\right)^{-1}\cdot\cos x = \mathbf{2}$$

10. $\lim\limits_{x\to+\infty}\ (2-x)\log x = -\infty$

11. $\lim\limits_{x\to 0}\dfrac{\log_3(1+2x)}{2x}$

$$\lim_{x\to 0}\ \log_3(1+2x)^{\frac{1}{2x}} \underset{2x=\frac{1}{y}}{\overset{=}{\downarrow}} \lim_{y\to\infty}\ \log_3\left(1+\frac{1}{y}\right)^{y} = \boldsymbol{\log_3 e}$$

12. $\lim\limits_{x\to 0}\dfrac{3^x-1}{2x}$ *cambio di variabile*: $3^x-1=y$ $x=\log_3(y+1)$

$$\lim_{y\to 0}\frac{y}{2\ \log_3(y+1)} = \frac{1}{2}\lim_{y\to 0}\frac{1}{\log_3(y+1)^{\frac{1}{y}}} \underset{y=\frac{1}{t}}{\overset{=}{\downarrow}} \frac{1}{2}\lim_{t\to\infty}\frac{1}{\log_3\left(1+\frac{1}{t}\right)^{t}} = \frac{1}{2}\,\frac{1}{\log_3 e} = \boldsymbol{\frac{ln\ e}{2}}$$

13. $\lim_{x\to 3} \frac{\log_3 x + \log_3 \frac{3}{x}}{x-2} = \frac{1+0}{3-2} = \mathbf{1}$

14. $\lim_{x\to -\infty} \frac{e^x}{x} = \lim_{x\to -\infty} e^x \cdot \frac{1}{x} = \mathbf{0}$

15. $\lim_{x\to +\infty} (1-x^2)\, e^x = -\infty$

16. $\lim_{x\to -2^-} \frac{4x+3}{x^2-4} = \lim_{x\to -2^-} \frac{4x+3}{(x+2)(x-2)} = -\infty$

17. $\lim_{x\to 3} \frac{x^3+x^2-12x}{x^3-3x^2+2x-6}$

$$\lim_{x\to 3} \frac{x\,(x^2+x-12)}{x^2(x-3)+2\,(x-3)} = \lim_{x\to 3} \frac{x\,(x-3)(x+4)}{(x-3)(x^2+2)} = \frac{3\cdot 7}{9+2} = \mathbf{\frac{21}{11}}$$

18. $\lim_{x\to +\infty} \frac{x^2-3x^4}{2x^2-x+4x^4} = \lim_{x\to +\infty} \frac{-3x^4}{4x^4} = -\mathbf{\frac{3}{4}}$