

51.  $\sqrt{1 + \log_{\sqrt{2}} x} = 3$

$$c.a.: \begin{cases} x > 0 \\ 1 + \log_{\sqrt{2}} x \geq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ \log_{\sqrt{2}} x \geq -1 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x \geq \frac{1}{\sqrt{2}} \end{cases} \Rightarrow x \geq \frac{1}{\sqrt{2}}$$

$$1 + \log_{\sqrt{2}} x = 9 \Rightarrow \log_{\sqrt{2}} x = 8 \Rightarrow x = \sqrt{2}^8 \Rightarrow x = 16$$

52.  $\log_{\frac{1}{3}} \log_{\frac{1}{3}} (5x + 9) = 0$

$$c.a.: \begin{cases} 5x + 9 > 0 \\ \log_{\frac{1}{3}} (5x + 9) > 0 \end{cases} \Rightarrow \begin{cases} x > -\frac{9}{5} \\ 5x + 9 \leq 1 \end{cases} \Rightarrow \begin{cases} x > -\frac{9}{5} \\ x \leq -\frac{8}{5} \end{cases} \Rightarrow -\frac{9}{5} < x \leq -\frac{8}{5}$$

$$\log_{\frac{1}{3}} (5x + 9) = 1 \Rightarrow 5x + 9 = \frac{1}{3} \Rightarrow 5x = -\frac{26}{3} \Rightarrow x = -\frac{26}{15}$$

53.  $(x^2 - 1) 2^{\log_2(x-2)} = \log_2 2^{x+1}$

$$2^{\log_2(x-2)} = x - 2 \quad \text{infatti, per la definizione di logaritmo: } a^b = c \Leftrightarrow \log_a c = b$$

$$\text{cioè: } 2^{\log_2(x-2)} = x - 2 \Leftrightarrow \log_2(x-2) = \log_2(x-2)$$

$$c.a.: x - 2 > 0 \Rightarrow x > 2: \quad (x^2 - 1)(x - 2) = x + 1$$

$$(x - 1)(x + 1)(x - 2) = x + 1 \Rightarrow x^2 - 3x + 2 = 1$$

$$x^2 - 3x + 1 = 0 \Rightarrow x_{1,2} = \frac{3 \pm \sqrt{9 - 4}}{2} = \begin{cases} \frac{3 + \sqrt{5}}{2} \\ \frac{3 - \sqrt{5}}{2} \end{cases} \text{ non acc.}$$

$$x = \frac{3 + \sqrt{5}}{2}$$

54.  $\ln^2 x + \ln x = 0$

$$c.a.: x > 0 \quad \text{Pongo: } t = \ln x \Rightarrow t^2 + t = 0 \Rightarrow t(t + 1) = 0$$

$$t = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$$

$$t = -1 \Rightarrow \ln x = -1 \Rightarrow x = \frac{1}{e}$$

55.  $\ln^3 x + 2 \ln^2 x - 3 \ln x = 0$

*c.a.:*  $x > 0$  Pongo:  $t = \ln x \Rightarrow t^3 + 2t^2 - 3t = 0 \Rightarrow t(t^2 + 2t - 3) = 0$

$$t_{1,2} = \frac{-1 \pm \sqrt{1+3}}{1} \begin{cases} 1 \\ -3 \end{cases}$$

$t = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$

$t = 1 \Rightarrow \ln x = 1 \Rightarrow x = e$

$t = -3 \Rightarrow \ln x = -3 \Rightarrow x = \frac{1}{e^3}$

56.  $|2 \ln x + 1| = 4$

*c.a.:*  $x > 0$ :  $2 \ln x + 1 = \pm 4$

$2 \ln x = 3 \Rightarrow \ln x = \frac{3}{2} \Rightarrow x = e^{\frac{3}{2}} \Rightarrow x = e\sqrt{e}$

$2 \ln x = -5 \Rightarrow \ln x = -\frac{5}{2} \Rightarrow x = e^{-\frac{5}{2}} \Rightarrow x = \frac{1}{e^2\sqrt{e}}$

57.  $\ln x = \log x$

*c.a.:*  $x > 0$  Applico la regola del cambiamento di base:

$\ln x = \frac{\ln x}{\ln 10} \Rightarrow (\ln 10 - 1) \ln x = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$

58.  $\ln \frac{x^2 - 1}{x} = \ln 2$

*c.a.:*  $\frac{x^2 - 1}{x} > 0 \Rightarrow \begin{cases} N > 0: x < -1 \vee x > 1 \\ D > 0: x > 0 \end{cases} \Rightarrow -1 < x < 0 \vee x > 1$

$\frac{x^2 - 1}{x} = 2 \Rightarrow \frac{x^2 - 1 - 2x}{x} = 0 \Rightarrow x^2 - 2x - 1 = 0 \Rightarrow$

$$x_{1,2} = \frac{1 \pm \sqrt{1+1}}{1} \begin{cases} 1 + \sqrt{2} \\ 1 - \sqrt{2} \end{cases}$$

$x = 1 + \sqrt{2}; \quad x = 1 - \sqrt{2}$

59.  $\ln(e^x + e) = 2$

c.a.:  $e^x + e > 0 \Rightarrow \forall x \in \mathbb{R}$

$e^x + e = e^2 \Rightarrow e^x = e^2 - e \Rightarrow x = \ln e(e - 1) \Rightarrow x = 1 + \ln(e - 1)$

60.  $\ln^2 x + 2 |\ln x| - 15 = 0$

c.a.:  $x > 0$  Pongo:  $t = |\ln x| \Rightarrow t^2 + 2t - 15 = 0 \Rightarrow t_{1,2} = \frac{-1 \pm \sqrt{1 + 15}}{1}$

$t = 3 \Rightarrow |\ln x| = 3 \Rightarrow \ln x = \pm 3 \Rightarrow x = e^{\pm 3} \Rightarrow x = e^3; x = \frac{1}{e^3}$

$t = -5 \Rightarrow |\ln x| = -5 \Rightarrow \text{imp.}$

61.  $\sqrt{\ln x} = 1 - \ln x$

c.a.:  $\begin{cases} x > 0 \\ \ln x \geq 0 \\ 1 - \ln x \geq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x \geq 1 \\ x \leq e \end{cases} \Rightarrow 1 \leq x \leq e$

pongo:  $\ln x = t$

$\sqrt{t} = 1 - t \Rightarrow t = 1 - 2t + t^2 \Rightarrow t^2 - 3t + 1 = 0$

$t_{1,2} = \frac{3 \pm \sqrt{9 - 4}}{2} \begin{cases} \frac{3 + \sqrt{5}}{2} \Rightarrow \ln x = \frac{3 + \sqrt{5}}{2} \text{ non acc.} \\ \frac{3 - \sqrt{5}}{2} \Rightarrow \ln x = \frac{3 - \sqrt{5}}{2} \Rightarrow \end{cases}$

$x = e^{\frac{3 - \sqrt{5}}{2}}$