

Equazioni esponenziali

48. $7^x = 9$
 $x = \log_7 9$

49. $10^x = \frac{1}{3}$
 $x = \log \frac{1}{3}$

50. $e^{2x} = 3$
 $2x = \ln 3 \Rightarrow x = \frac{1}{2} \ln 3 \Rightarrow x = \ln \sqrt{3}$

51. $e^{|x|} = 2$
 $|x| = \ln 2 \Rightarrow x = \pm \ln 2$

52. $e^{2x-1} \cdot e^{-x+7} = 3$
 $e^{x+6} = 3 \Rightarrow x + 6 = \ln 3 \Rightarrow x = \ln 3 - 6$

53. $(2e^{-x} - 3)(e^{3x} + 5) = 0$
 $2e^{-x} - 3 = 0 \Rightarrow e^{-x} = \frac{3}{2} \Rightarrow -x = \ln \frac{3}{2} \Rightarrow x = \ln \frac{2}{3}$

54. $3^{1-x} - 3^{2+x} = 1$
 Pongo: $3^x = t \Rightarrow \frac{3}{t} - 9t = 1 \Rightarrow 9t^2 + t - 3 = 0 \Rightarrow t_{1,2} = \frac{-1 \pm \sqrt{109}}{18}$
 Non accettabile la soluzione negativa, visto che $3^x > 0 \forall x$, perciò: $x = \log_3 \frac{-1 + \sqrt{109}}{18}$

55. $\frac{3^{4x-2}}{3^{x-2}} - 2 \cdot 3^{2x+1} - \frac{57}{3^{1-x}} + 84 = 0$
 $3^{3x} - 6 \cdot 3^{2x} - 19 \cdot 3^x + 84 = 0$
 Pongo: $3^x = t \Rightarrow t^3 - 6t^2 - 19t + 84 = 0$
 $(t - 3)(t^2 - 3t - 28) = 0 \Rightarrow$
 $t = 3 \Rightarrow 3^x = 3 \Rightarrow x = 1$
 $t = 7 \Rightarrow 3^x = 7 \Rightarrow x = \log_3 7$
 $t = -4 \Rightarrow 3^x = -4 \Rightarrow \text{imp.}$

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$$56. \quad |3^{2x} - 2| = 3^x$$

$$3^{2x} - 2 = -3^x \Rightarrow 3^{2x} + 3^x - 2 = 0 \Rightarrow (3^x + 2)(3^x - 1) = 0 \Rightarrow$$

$$3^x - 1 = 0 \Rightarrow x = 0$$

$$3^{2x} - 2 = 3^x \Rightarrow 3^{2x} - 3^x - 2 = 0 \Rightarrow (3^x - 2)(3^x + 1) = 0 \Rightarrow$$

$$3^x - 2 = 0 \Rightarrow x = \log_3 2$$

$$57. \quad 3 \cdot 2^{x \log_2 9} - 2^{x \log_2 3} - 2 = 0$$

$$3 \cdot 2^{\log_2 9^x} - 2^{\log_2 3^x} - 2 = 0 \Rightarrow 3 \cdot 9^x - 3^x - 2 = 0$$

Pongo: $3^x = t \Rightarrow 3t^2 - t - 2 = 0 \Rightarrow$

$$t = 1 \Rightarrow 3^x = 1 \Rightarrow x = 0$$

$$t = -\frac{2}{3} \Rightarrow 3^x = -\frac{2}{3} \Rightarrow \text{imp.}$$

$$58. \quad 3^{3x} - 3^{2x+1} + 2 = 0$$

Pongo: $3^x = t \Rightarrow t^3 - 3t^2 + 2 = 0 \Rightarrow (t - 1)(t^2 - 2t - 2) = 0$

$$t = 1 \Rightarrow 3^x = 1 \Rightarrow x = 0$$

$$t = 1 - \sqrt{3} \Rightarrow 3^x = 1 - \sqrt{3} \Rightarrow \text{imp.}$$

$$t = 1 + \sqrt{3} \Rightarrow 3^x = 1 + \sqrt{3} \Rightarrow x = \log_3(1 + \sqrt{3})$$

$$59. \quad 2^{3x} - 2^{2x+1} - 23 \cdot 2^x + 60 = 0$$

Pongo: $2^x = t \Rightarrow t^3 - 2t^2 - 23t + 60 = 0 \Rightarrow (t - 3)(t^2 + t - 20) = 0$

$$t = 3 \Rightarrow 2^x = 3 \Rightarrow x = \log_2 3$$

$$t = 4 \Rightarrow 2^x = 4 \Rightarrow x = 2$$

$$t = -5 \Rightarrow 2^x = -5 \Rightarrow \text{imp.}$$

$$60. \quad 2^{2x} - 5^x - 4^{x-1} + 25^{\frac{x}{2}-1} = 0$$

$$4^x - 5^x - \frac{4^x}{4} + \frac{5^x}{25} = 0 \Rightarrow 4^x \left(1 - \frac{1}{4}\right) - 5^x \left(1 - \frac{1}{25}\right) = 0 \Rightarrow$$

$$\frac{3}{4} 4^x = \frac{24}{25} 5^x \Rightarrow \left(\frac{4}{5}\right)^x = \frac{24}{25} \cdot \frac{4}{3} \Rightarrow \left(\frac{4}{5}\right)^x = \frac{32}{25} \Rightarrow x = \log_{\frac{4}{5}} \frac{32}{25}$$

$$61. \quad \frac{1}{3^x - 1} + \frac{2}{3^x + 1} = 2$$

$$\text{Pongo: } 3^x = t \Rightarrow \frac{1}{t-1} + \frac{2}{t+1} - 2 = 0 \Rightarrow t + 1 + 2t - 2 - 2t^2 + 2 = 0$$

$$(t \neq \pm 1) \quad 2t^2 - 3t - 1 = 0 \Rightarrow t_{1,2} = \frac{3 \pm \sqrt{17}}{4}$$

$$t = \frac{3 - \sqrt{17}}{4} \Rightarrow 3^x = \frac{3 - \sqrt{17}}{4} \Rightarrow \text{imp.}$$

$$t = \frac{3 + \sqrt{17}}{4} \Rightarrow 3^x = \frac{3 + \sqrt{17}}{4} \Rightarrow x = \log_3 \frac{3 + \sqrt{17}}{4}$$

$$62. \quad \frac{9}{3^x - 3} - \frac{4}{3^x - 6} = \frac{9^x - 56}{9^x - 3^{x+2} + 18}$$

$$\text{Pongo: } 3^x = t \Rightarrow \frac{9}{t-3} - \frac{4}{t-6} = \frac{t^2 - 56}{t^2 - 9t + 18}$$

$$9t - 54 - 4t + 12 = t^2 - 56 \quad t \neq 3; t \neq 6$$

$$t^2 - 5t - 14 = 0 \Rightarrow t = 7; t = -2$$

$$t = -2 \Rightarrow 3^x = -2 \Rightarrow \text{imp.}$$

$$t = 7 \Rightarrow 3^x = 7 \Rightarrow x = \log_3 7$$

$$63. \quad e^{2x} - 5e^x - 36 = 0$$

$$\text{Pongo: } e^x = t \Rightarrow t^2 - 5t - 36 = 0 \Rightarrow t_{1,2} = \frac{5 \pm 13}{2}$$

$$t = -4 \Rightarrow e^x = -4 \Rightarrow \text{imp.}$$

$$t = 9 \Rightarrow e^x = 9 \Rightarrow x = \ln 9$$

$$64. \quad e^{2x} - 4e^x - 29 - 24e^{-x} = 0$$

$$\text{Pongo: } e^x = t \Rightarrow t^2 - 4t - 29 - \frac{24}{t} = 0 \Rightarrow t^3 - 4t^2 - 29t - 24 = 0$$

$$(t + 1)(t^2 - 5t - 24) = 0$$

$$t = -1 \Rightarrow e^x = -1 \Rightarrow \text{imp.}$$

$$t = -3 \Rightarrow e^x = -3 \Rightarrow \text{imp.}$$

$$t = 8 \Rightarrow e^x = 8 \Rightarrow x = \ln 8$$

Equazioni esponenziali

65. $e^{x+1} - 10^4 \cdot e^{-(x+1)} - 45 = 0$

Pongo: $e^{x+1} = t \Rightarrow t - \frac{10^4}{t} - 45 = 0 \Rightarrow t^2 - 45t - 10^4 = 0$

$$t_{1,2} = \frac{45 \pm 205}{2} \begin{cases} 125 \\ -80 \end{cases}$$

$t = -80 \Rightarrow e^{x+1} = -80 \Rightarrow \text{imp.}$

$t = 125 \Rightarrow e^{x+1} = 125 \Rightarrow x + 1 = \ln 125 \Rightarrow x = \ln 125 - 1$

66. $e^{|x|-1} = 3$

$|x| - 1 = \ln 3 \Rightarrow |x| = 1 + \ln 3 \Rightarrow x = \pm (1 + \ln 3)$

67. $\frac{1}{|e^x - 1|} = 1$

c.a.: $e^x - 1 \neq 0 \Rightarrow x \neq 0$

$|e^x - 1| = 1 \Rightarrow e^x - 1 = \pm 1$

$e^x - 1 = 1 \Rightarrow e^x = 2 \Rightarrow x = \ln 2$

$e^x - 1 = -1 \Rightarrow e^x = 0 \Rightarrow \text{imp.}$