

Disequazioni esponenziali

83. $4^{x^2-2x-3} > 2$

$$2^{2(x^2-2x-3)} > 2 \Rightarrow 2x^2 - 4x - 6 > 1 \Rightarrow 2x^2 - 4x - 7 > 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4+14}}{2} = \frac{2 \pm 3\sqrt{2}}{2} \Rightarrow x < \frac{2-3\sqrt{2}}{2} \vee x > \frac{2+3\sqrt{2}}{2}$$

84. $2^{x+2} + 4^{x+2} > 272$

Pongo: $2^x = t \Rightarrow 4t + 16t^2 - 272 > 0 \Rightarrow 4t^2 + t - 68 > 0$

$$t_{1,2} = \frac{-1 \pm \sqrt{1+1088}}{8} = \left\langle \begin{array}{l} 4 \\ -\frac{17}{4} \end{array} \right. \Rightarrow t < -\frac{17}{4} \vee t > 4 \Rightarrow x > 2$$

85. $2^{x+3} + 7^x \geq 9 \cdot 7^x - 20 \cdot 2^x$

$$2^x \cdot 2^3 + 2^2 \cdot 5 \cdot 2^x + 7^x - 9 \cdot 7^x \geq 0 \Rightarrow 2^x(2^3 + 5 \cdot 2^2) + 7^x(1-9) \geq 0 \Rightarrow$$

$$2^x \cdot 28 - 7^x \cdot 8 \geq 0 \Rightarrow \frac{2^x}{7^x} \geq \frac{8}{28} \Rightarrow \left(\frac{2}{7}\right)^x \geq \frac{2}{7} \Rightarrow x \leq 1$$

86. $3^x + \frac{1}{3^{x+1}} > \frac{28}{9}$

Pongo: $3^x = t \Rightarrow t + \frac{1}{3t} > \frac{28}{9} \Rightarrow \frac{9t^2 + 3 - 28t}{9t} > 0 \Rightarrow 9t^2 - 28t + 3 > 0$

$$t_{1,2} = \frac{14 \pm \sqrt{196-27}}{9} = \left\langle \begin{array}{l} 3 \\ \frac{1}{9} \end{array} \right. \Rightarrow t < \frac{1}{9} \vee t > 3 \Rightarrow x < -2 \vee x > 1$$

87. $3^{2+x-2x^2} < 3^{(2-x)^2}$

$$2+x-2x^2 < 4-4x+x^2 \Rightarrow -3x^2+5x-2 < 0 \Rightarrow 3x^2-5x+2 > 0 \Rightarrow$$

$$x_{1,2} = \frac{5 \pm \sqrt{25-24}}{6} = \left\langle \begin{array}{l} 1 \\ \frac{2}{3} \end{array} \right. \Rightarrow x < \frac{2}{3} \vee x > 1$$

88. $\frac{25^{1-x}}{(5^{x-2})^{3+x}} < \frac{25^{-2x} \cdot 5^{-9}}{(5^{2-x})^{2x-3}}$

$$\frac{5^{2-2x}}{5^{x^2+x-6}} < \frac{5^{-4x} \cdot 5^{-9}}{5^{-2x^2+7x-6}} \Rightarrow 5^{2-2x-x^2-x+6} < 5^{-4x-9+2x^2-7x+6}$$

$$-x^2-3x+8 < 2x^2-11x-3 \Rightarrow 3x^2-8x-11 > 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{16+33}}{3} = \left\langle \begin{array}{l} \frac{11}{3} \\ -1 \end{array} \right. \Rightarrow x < -1 \vee x > \frac{11}{3}$$

$$89. \quad \sqrt{8} \cdot 2^x - 2^{2x+\frac{1}{2}} : 2 < (2+2^x) : \sqrt{2}$$

$$\text{Pongo: } 2^x = t \Rightarrow 2\sqrt{2}t - \frac{\sqrt{2}}{2}t^2 < \frac{2}{\sqrt{2}} + \frac{t}{\sqrt{2}}$$

$$2\sqrt{2}t - \frac{\sqrt{2}}{2}t^2 < \sqrt{2} + \frac{\sqrt{2}}{2}t \Rightarrow 2t - \frac{1}{2}t^2 < 1 + \frac{1}{2}t$$

$$-t^2 + 3t - 2 < 0 \Rightarrow t^2 - 3t + 2 > 0 \Rightarrow t < 1 \vee t > 2 \Rightarrow \boxed{x < 0 \vee x > 1}$$

$$90. \quad 2^{x+1} + \frac{8}{2^x} \geq 17$$

$$\text{Pongo: } 2^x = t \Rightarrow 2t + \frac{8}{t} \geq 17 \Rightarrow 2t^2 - 17t + 8 \geq 0$$

$$t_{1,2} = \frac{17 \pm \sqrt{289 - 64}}{4} = \left\langle \frac{8}{1} \right\rangle \Rightarrow t \leq \frac{1}{2} \vee t \geq 8 \Rightarrow \boxed{x \leq -1 \vee x \geq 3}$$

$$91. \quad \frac{\left(\frac{4}{3}\right)^x - \frac{16}{9}}{3^{2x} - 4 \cdot 3^x + 3} < 0$$

$$N > 0: \left(\frac{4}{3}\right)^x > \frac{16}{9} \Rightarrow x > 2$$

$$D > 0: \text{ Pongo: } 3^x = t \Rightarrow t^2 - 4t + 3 > 0 \Rightarrow t_{1,2} = \frac{2 \pm \sqrt{4-3}}{1} = \left\langle \frac{3}{1} \right\rangle \Rightarrow$$

$$t < 1 \vee t > 3 \Rightarrow x < 0 \vee x > 1 \quad \boxed{x < 0 \vee 1 < x < 2}$$

$$92. \quad \frac{4^x + 2^{x+1} - 80}{8 \cdot 2^x - 2} < 0$$

$$N > 0: \text{ Pongo: } 2^x = t \Rightarrow t^2 + 2t - 80 > 0 \Rightarrow t_{1,2} = \frac{-1 \pm 9}{1} = \left\langle \frac{8}{-10} \right\rangle \Rightarrow$$

$$t < -10 \vee t > 8 \Rightarrow x > 3$$

$$D > 0: 2^{x+3} > 2^1 \Rightarrow x+3 > 1 \Rightarrow x > -2 \quad \boxed{-2 < x < 3}$$

$$93. \quad \frac{\sqrt[4]{3^{x^2}}}{3^x} > \frac{\sqrt{3}}{9}$$

$$\frac{3^{\frac{x^2}{4}}}{3^x} > \frac{3^{\frac{1}{2}}}{9} \Rightarrow 3^{\frac{x^2}{4}-x} > 3^{\frac{1}{2}-2} \Rightarrow \frac{x^2}{4} - x > \frac{1}{2} - 2 \Rightarrow x^2 - 4x + 6 > 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4-6}}{1} \quad \boxed{\forall x \in \mathbb{R}}$$

Disequazioni esponenziali

$$94. \frac{2^{5x+1} \cdot 16^{x-1}}{8^{x+1}} < 4^{2x-1}$$

$$\frac{2^{5x+1} \cdot 2^{4x-4}}{2^{3x+3}} < 2^{4x-2} \Rightarrow 2^{5x+1+4x-4-3x-3} < 2^{4x-2} \Rightarrow$$

$$6x - 6 < 4x - 2 \Rightarrow 2x < 4 \Rightarrow x < 2$$

$$95. (2 + 2^x)^2 - (2 - 2^x)^2 \leq 4$$

$$\text{Pongo: } 2^x = t \Rightarrow (2 + t)^2 - (2 - t)^2 \leq 4 \Rightarrow 8t \leq 4 \Rightarrow t \leq \frac{1}{2} \Rightarrow x \leq -1$$

$$96. 3 \cdot 3^{2x} + 4 \cdot 6^x + 2^{2x} < 0$$

impossibile

$$97. 4 \cdot \left(\frac{3}{2}\right)^{2x} + 15 \cdot \left(\frac{3}{2}\right)^x < 19$$

$$\text{Pongo: } \left(\frac{3}{2}\right)^x = t \Rightarrow 4t^2 + 15t - 19 < 0 \Rightarrow t_{1,2} = \frac{-15 \pm 23}{8} = \begin{cases} -\frac{19}{4} \\ 1 \end{cases} \Rightarrow$$

$$-\frac{19}{4} < t < 1 \Rightarrow x < 0$$

$$98. \frac{3^{2x} - 3^{x+1}}{9^x - 1} \leq 0$$

$$N \geq 0: 3^{2x} \geq 3^{x+1} \Rightarrow 2x \geq x+1 \Rightarrow x \geq 1$$

$$D > 0: 9^x > 1 \Rightarrow x > 0 \quad \quad \quad 0 < x \leq 1$$

$$99. 3^{2x} - 10 \cdot 3^x + 9 < 0$$

$$\text{Pongo: } 3^x = t \Rightarrow t^2 - 10t + 9 < 0 \Rightarrow 1 < t < 9 \Rightarrow 0 < x < 2$$

$$100. 5^{x+1} + 5^{1-x} - 26 > 0$$

$$\text{Pongo: } 5^x = t \Rightarrow 5t + \frac{5}{t} - 26 > 0 \Rightarrow 5t^2 - 26t + 5 > 0 \Rightarrow$$

$$t_{1,2} = \frac{13 \pm \sqrt{169 - 25}}{5} = \begin{cases} 5 \\ \frac{1}{5} \end{cases} \Rightarrow t < \frac{1}{5} \vee t > 5 \Rightarrow x < -1 \vee x > 1$$

Disequazioni esponenziali

101. $5^x (5^x + 1) > 5^{x+2} + 25$

Pongo: $5^x = t \Rightarrow t(t+1) > 25t + 25 \Rightarrow (t-25)(t+1) > 0 \Rightarrow$

$t - 25 > 0 \Rightarrow 5^x > 25 \Rightarrow x > 2$

102. $2^{2x+1} - 9 \cdot 2^x + 4 < 0$

Pongo: $2^x = t \Rightarrow 2t^2 - 9t + 4 < 0 \Rightarrow t_{1,2} = \frac{9 \pm \sqrt{81 - 32}}{4} = \left\langle \begin{array}{l} 4 \\ \frac{1}{2} \end{array} \right\rangle \Rightarrow$

$\frac{1}{2} < t < 4 \Rightarrow -1 < x < 2$