

21. $\sqrt{3} \operatorname{sen} 2x - \cos 2x \geq 0$

Trattandosi di una disequazione omogenea di primo grado, si procede raccogliendo il termine $\cos 2x$:

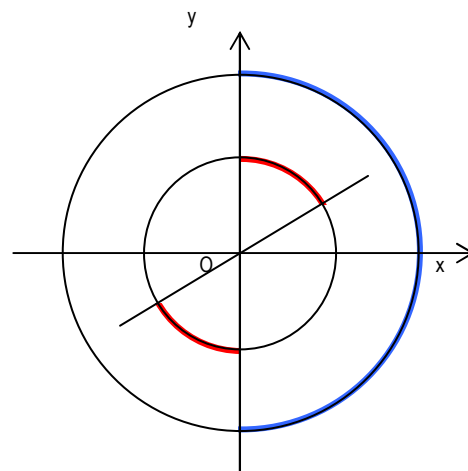
$$\cos 2x (\sqrt{3} \operatorname{tg} 2x - 1) \geq 0$$

$$\cos 2x \geq 0$$

$$\operatorname{tg} 2x \geq \frac{\sqrt{3}}{3}$$

$$\frac{\pi}{6} + 2k\pi \leq 2x \leq \frac{7}{6}\pi + 2k\pi$$

$$\frac{\pi}{12} + k\pi \leq x \leq \frac{7}{12}\pi + k\pi$$

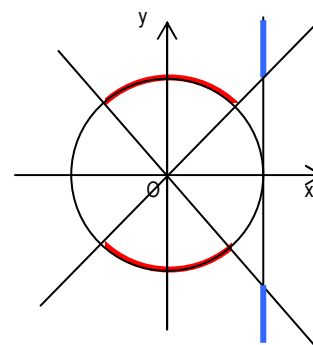


22. $\operatorname{sen}^2 x - \cos^2 x > 0$

Disequazione omogenea di secondo grado. Dividiamo entrambi i membri per $\cos^2 x$:

$$\operatorname{tg}^2 x - 1 > 0 \Rightarrow \operatorname{tg} x < -1 \vee \operatorname{tg} x > 1$$

$$\frac{\pi}{4} + k\pi < x < \frac{3}{4}\pi + k\pi$$



Soluzione alternativa:

$$-(-\operatorname{sen}^2 x + \cos^2 x) > 0$$

$$\cos^2 x - \operatorname{sen}^2 x < 0$$

$$\cos 2x < 0 \Rightarrow \frac{\pi}{2} + 2k\pi < 2x < \frac{3}{2}\pi + 2k\pi \Rightarrow$$

$$\frac{\pi}{4} + k\pi < x < \frac{3}{4}\pi + k\pi$$

23. $\cos^2 x - 2\sqrt{3} \cos x \operatorname{sen} x - \operatorname{sen}^2 x > 0$

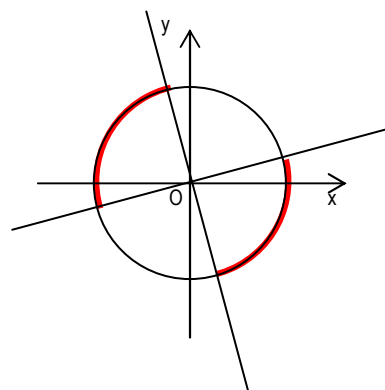
Disequazione omogenea di secondo grado. Dividiamo entrambi i membri per $\cos^2 x$:

$$\operatorname{tg}^2 x - 2\sqrt{3} \operatorname{tg} x - 1 < 0$$

$$\operatorname{tg} x = -\sqrt{3} \pm \sqrt{3+1} = -\sqrt{3} \pm 2$$

$$-\sqrt{3} - 2 < \operatorname{tg} x < -\sqrt{3} + 2$$

$$-\frac{5}{12}\pi + k\pi \leq x \leq \frac{\pi}{12} + k\pi$$



24. $\text{sen}^2 x + \text{sen} x \cos x < 0$

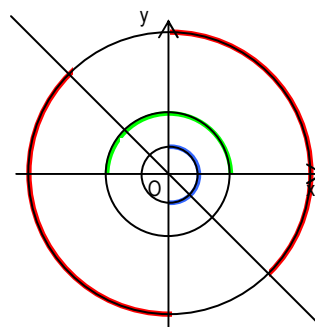
$\text{sen} x \cos x (\text{tg} x + 1) < 0$

$\text{sen} x > 0$

$\cos x > 0$

$\text{tg} x > -1$

$\frac{3}{4}\pi + k\pi < x < \pi + k\pi$



25. $6 \text{sen}^2 x - 2\sqrt{3} \text{sen} x \cos x < 3$

Disequazione riconducibile a omogenea di secondo grado: $6 \text{sen}^2 x - 2\sqrt{3} \text{sen} x \cos x < 3 (\text{sen}^2 x + \cos^2 x)$

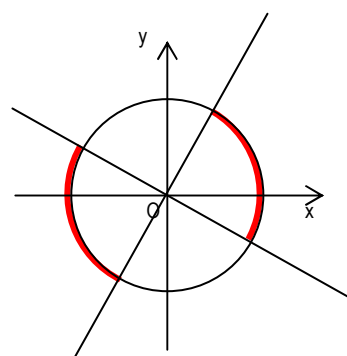
$3 \text{sen}^2 x - 2\sqrt{3} \text{sen} x \cos x - 3 \cos^2 x < 0$

Divido per $\cos^2 x$: $3 \text{tg}^2 x - 2\sqrt{3} \text{tg} x - 3 < 0$

$\text{tg} x = \frac{\sqrt{3} \pm \sqrt{3+9}}{3} = \begin{cases} \sqrt{3} \\ -\frac{\sqrt{3}}{3} \end{cases}$

$-\frac{\sqrt{3}}{3} < \text{tg} x < \sqrt{3}$

$-\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi$



26. $\text{sen}^4 x - \cos^4 x < 0$

$(\text{sen}^2 x - \cos^2 x) (\text{sen}^2 x + \cos^2 x) < 0$ che, per la prima relazione fondamentale, diventa:

$\text{sen}^2 x - \cos^2 x < 0$ disequazione omogenea di secondo grado, perciò divido per $\cos^2 x$:

$\text{tg}^2 x - 1 < 0 \Rightarrow -1 < \text{tg} x < 1$

$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi$

