

$$\begin{aligned}
 1. \quad & 3x(7-x) - 4(x+5)(3x-1) + 7(2-x)(x+2) + 22x^2 + 35x = \\
 & = 21x - 3x^2 - 4(3x^2 - x - 15x - 5) + 7(4 - x^2) + 22x^2 + 35x = \\
 & = 21x - 3x^2 - 12x^2 + 4x + 60x + 20 + 28 - 7x^2 + 22x^2 + 35x = 48
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & (x-2)^3 - (2x-1)^2 - x^2(x-10) = \\
 & = x^3 - 6x^2 + 12x - 8 - (4x^2 - 4x + 1) - x^3 + 10x^2 = \\
 & = -6x^2 + 12x - 8 - 4x^2 + 4x - 1 + 10x^2 = 16x - 9
 \end{aligned}$$

3. Risolvi le seguenti espressioni con le frazioni algebriche:

$$\begin{aligned}
 a. \quad & \left(3x - \frac{y^2}{2x}\right) \cdot \left(3x - \frac{3x^2 - 2y^2}{x+y}\right) : \left(2 + \frac{x}{x+y}\right) : (6x^2 - y^2) = \\
 & = \left(\frac{6x^2 - y^2}{2x}\right) \cdot \left(\frac{3x^2 + 3xy - 3x^2 + 2y^2}{x+y}\right) : \left(\frac{2x + 2y + x}{x+y}\right) \cdot \frac{1}{6x^2 - y^2} = \\
 & = \frac{6x^2 - y^2}{2x} \cdot \frac{y(3x + 2y)}{x+y} \cdot \frac{x+y}{3x+2y} \cdot \frac{1}{6x^2 - y^2} = \frac{y}{2x}
 \end{aligned}$$

$$\begin{aligned}
 b. \quad & \left[\left(\frac{2}{x-5y} - \frac{2}{x+5y}\right) : \left(\frac{4}{x+5y} - \frac{4}{5y-x}\right) - \frac{y}{x}\right] : (3y) = \\
 & = \left[\left(\frac{2x + 10y - 2x + 10y}{(x-5y)(x+5y)}\right) : \left(\frac{20y - 4x - 4x - 20y}{(x+5y)(5y-x)}\right) - \frac{y}{x}\right] : (3y) = \\
 & = \left[\left(\frac{20y}{(x-5y)(x+5y)} \cdot \frac{-(x-5y)(x+5y)}{-8x}\right) - \frac{y}{x}\right] \cdot \frac{1}{3y} = \\
 & = \left(\frac{5y}{2x} - \frac{y}{x}\right) \cdot \frac{1}{3y} = \frac{5y - 2y}{2x} \cdot \frac{1}{3y} = \frac{3y}{2x} \cdot \frac{1}{3y} = \frac{1}{2x}
 \end{aligned}$$

4. Risolvi le seguenti equazioni:

$$\begin{aligned}
 a. \quad & \frac{1}{7} \left(\frac{1+3x}{4} - x\right) = \frac{1-x}{28} + 2 + \frac{2-x}{3} \\
 & \frac{1}{7} \left(\frac{1+3x-4x}{4}\right) = \frac{1-x}{28} + 2 + \frac{2-x}{3} \\
 & \frac{1-x}{28} = \frac{1-x}{28} + 2 + \frac{2-x}{3} \\
 & 0 = 2 + \frac{2-x}{3} \\
 & 6 + 2 - x = 0 \quad \Rightarrow \quad x = 8
 \end{aligned}$$

$$b. \quad \frac{x+3}{2x+1} - \frac{2x+1}{2x-1} - \frac{2x^2-3x+5}{1-4x^2} = 0$$

$$\frac{x+3}{2x+1} - \frac{2x+1}{2x-1} + \frac{2x^2-3x+5}{(2x-1)(2x+1)} = 0$$

$$\frac{2x^2 - x + 6x - 3 - 4x^2 - 4x - 1 + 2x^2 - 3x + 5}{(2x-1)(2x+1)} = 0$$

$$C.A.: x \neq \pm \frac{1}{2}$$

$$-2x + 1 = 0$$

$$x = \frac{1}{2} \text{ non accettabile per C.A.} \Rightarrow \nexists x \in R$$

$$c. \quad \frac{2x}{4-x} + \frac{1}{2x} = \frac{1-x^2}{2x^2-8x} - \frac{3}{2}$$

$$-\frac{2x}{x-4} + \frac{1}{2x} = \frac{1-x^2}{2x(x-4)} - \frac{3}{2}$$

$$\frac{-4x^2 + x - 4 - 1 + x^2 + 3x^2 - 12x}{2x(x-4)} = 0$$

$$C.A.: x \neq 0, x \neq 4$$

$$-11x = 5 \Rightarrow x = -\frac{5}{11} \text{ acc.}$$

5. Risolvi i seguenti sistemi, il primo con tutti e quattro i metodi algebrici, il secondo con il metodo che preferisci:

$$a. \quad \begin{cases} x + \frac{1}{2}y = \frac{7}{2} \\ \frac{1}{4}x - y = 2 \end{cases} \Rightarrow \begin{cases} 2x + y = 7 \\ x - 4y = 8 \end{cases}$$

$$\text{CRAMER:} \quad D = \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} = -8 - 1 = -9$$

$$D_x = \begin{vmatrix} 7 & 1 \\ 8 & -4 \end{vmatrix} = -28 - 8 = -36$$

$$D_y = \begin{vmatrix} 2 & 7 \\ 1 & 8 \end{vmatrix} = 16 - 7 = 9$$

$$\begin{cases} x = \frac{D_x}{D} = \frac{-36}{-9} = 4 \\ y = \frac{D_y}{D} = \frac{9}{-9} = -1 \end{cases}$$

$$\text{SOSTITUZIONE: } \begin{cases} x = 4y + 8 \\ 2(4y + 8) + y = 7 \end{cases} \Rightarrow \begin{cases} x = 4y + 8 \\ 8y + 16 + y = 7 \end{cases}$$

$$\begin{cases} x = 4y + 8 \\ 9y = -9 \end{cases} \Rightarrow \begin{cases} x = 4(-1) + 8 \\ y = -1 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = -1 \end{cases}$$

$$\text{RIDUZIONE: } \begin{cases} 8x + 4y = 28 \\ x - 4y = 8 \end{cases} \Rightarrow \begin{array}{l} 9x = 36 \\ x = 4 \end{array}$$

$$\begin{cases} 2x + y = 7 \\ -2x + 8y = -16 \end{cases} \Rightarrow \begin{array}{l} 9y = -9 \\ y = -1 \end{array}$$

$$\text{CONFRONTO: } \begin{cases} y = 7 - 2x \\ y = \frac{x - 8}{4} \end{cases} \Rightarrow 7 - 2x = \frac{x - 8}{4}$$

$$28 - 8x = x - 8$$

$$x = 4$$

$$\begin{cases} x = \frac{7 - y}{2} \\ x = 4y + 8 \end{cases} \Rightarrow \frac{7 - y}{2} = 4y + 8$$

$$7 - y = 8y + 16$$

$$y = -1$$

$$\text{b. } \begin{cases} 5x - y = -5 \\ \frac{y - 2}{y + 7} = \frac{2}{3} \end{cases} \Rightarrow \begin{cases} 5x - y = -5 \\ \frac{3y - 6}{3(y + 7)} = \frac{2y + 14}{3(y + 7)} \end{cases} \quad \text{C.A.: } y \neq -7$$

$$\begin{cases} 5x - y = -5 \\ y = 20 \end{cases} \Rightarrow \begin{cases} 5x - 20 = -5 \\ y = 20 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 20 \end{cases}$$