

$$1. \quad \left(1 - \frac{2}{3}x\right)\left(\frac{2}{3}x + 1\right) + 4\left(\frac{1}{3}x - a\right)\left(\frac{1}{3}x + a\right) + (2a - 1)(2a + 1) =$$

$$= 1 - \frac{4}{9}x^2 + 4\left(\frac{1}{9}x^2 - a^2\right) + 4a^2 - 1 = 1 - \frac{4}{9}x^2 + \frac{4}{9}x^2 - 4a^2 + 4a^2 - 1 = 0$$

$$2. \quad \left[\left(\frac{x^3}{2} - \frac{1}{3}\right)^2 - \left(\frac{x^3}{2} - 1\right)\left(\frac{x^3}{2} + 1\right) + \frac{8}{9}\right]^3 =$$

$$= \left[\frac{1}{4}x^6 - \frac{1}{3}x^3 + \frac{1}{9} - \left(\frac{x^6}{4} - 1\right) + \frac{8}{9}\right]^3 = \left[\frac{1}{4}x^6 - \frac{1}{3}x^3 + \frac{1}{9} - \frac{1}{4}x^6 + 1 + \frac{8}{9}\right]^3 =$$

$$= \left(-\frac{1}{3}x^3 + 2\right)^3 = -\frac{1}{27}x^9 + \frac{2}{3}x^6 - 4x^3 + 8$$

3. Scomponi i seguenti polinomi:

$$16x^3y + 2y = 2y(8x^3 + 1) = 2y(2x + 1)(4x^2 - 2x + 1)$$

$$9a^3 - 6a^2 + a = a(9a^2 - 6a + 1) = a(3a - 1)^2$$

$$y^2 - 5y + 4 = (y - 4)(y - 1)$$

$$\frac{x^3}{4} - x^5 = x^3\left(\frac{1}{4} - x^2\right) = x^3\left(\frac{1}{2} - x\right)\left(\frac{1}{2} + x\right)$$

$$a^4b^3 - a^2b^5 = a^2b^3(a^2 - b^2) = a^2b^3(a - b)(a + b)$$

$$4. \quad \left(\frac{3}{x+2} - \frac{2}{x+3}\right) : \left(1 - \frac{6}{x^2 + 5x + 6}\right)$$

$$= \frac{3(x+3) - 2(x+2)}{(x+2)(x+3)} : \frac{x^2 + 5x + 6 - 6}{x^2 + 5x + 6} =$$

$$= \frac{3x + 9 - 2x - 4}{(x+2)(x+3)} : \frac{x^2 + 5x}{(x+2)(x+3)} =$$

$$= \frac{x + 5}{(x+2)(x+3)} \cdot \frac{(x+2)(x+3)}{x(x+5)} = \frac{1}{x}$$

$$5. \quad \left(\frac{x}{y} - \frac{y}{x}\right) : \left(\frac{2}{x} + \frac{2}{y}\right)^2$$

$$= \left(\frac{x^2 - y^2}{xy}\right) : \left(\frac{2y + 2x}{xy}\right)^2 = \frac{(x-y)(x+y)}{xy} \cdot \frac{x^2y^2}{4(x+y)^2} = \frac{xy(x-y)}{4(x+y)}$$

$$6. \quad \frac{a^3 - 1}{a^2 - 2a + 1} \cdot \frac{a - 1}{a^2 + a + 1}$$

$$= \frac{(a-1)(a^2 + a + 1)}{(a-1)^2} \cdot \frac{a-1}{a^2 + a + 1} = 1$$