

$$\begin{aligned}
 1. \quad & 5(x-1)^2 - x^3 \geq x^2(5-x) - 4x \\
 & 5(x^2 - 2x + 1) - x^3 \geq 5x^2 - x^3 - 4x \\
 & 5x^2 - 10x + 5 - x^3 \geq 5x^2 - x^3 - 4x \\
 & -6x \geq -5
 \end{aligned}$$

$$x \leq \frac{5}{6}$$

$$\begin{aligned}
 2. \quad & \frac{x-5}{3} + \frac{1}{6}x - \frac{1}{2}(x-1)(x+1) \geq \frac{5-x^2}{2} + \frac{1}{3} \\
 & \frac{2x-10+x-3(x^2-1)}{6} \geq \frac{15-3x^2+2}{6} \\
 & 2x-10+x-3x^2+3 \geq 15-3x^2+2 \\
 & 3x \geq 24
 \end{aligned}$$

$$x \geq 8$$

$$3. \quad \begin{cases} 5x - 7 > 3x + 1 \\ 6(x + 1) < 3(x - 5) \\ 2(2x - 5) - 3(x - 4) < 2x \end{cases}$$

$$\begin{cases} 2x > 8 \\ 6x + 6 < 3x - 15 \\ 4x - 10 - 3x + 12 < 2x \end{cases}$$

$$\begin{cases} x > 4 \\ 3x < -21 \\ -x < -2 \end{cases}$$

$$\begin{cases} x > 4 \\ x < -7 \\ x > 2 \end{cases}$$

imp.

$$\begin{aligned}
 4. \quad & \frac{x+5}{x} > \frac{2x+5}{2x+1} \\
 & \frac{2x^2 + 10x + x + 5 - 2x^2 - 5x}{x(2x+1)} > 0
 \end{aligned}$$

$$\frac{(2x+1)(x+5) - x(2x+5)}{x(2x+1)} > 0$$

$$\frac{6x+5}{x(2x+1)} > 0$$

$$N > 0: 6x + 5 > 0 \Rightarrow x > -\frac{5}{6}$$

$$D_1 > 0: x > 0$$

$$D_2 > 0: 2x + 1 > 0 \Rightarrow x > -\frac{1}{2}$$

$$-\frac{5}{6} < x < -\frac{1}{2} \vee x > 0$$

5. $|x + 4| + |x - 3| = 5$

$$\begin{array}{c|c|c} -4 & & 3 \\ \hline -x - 4 & | & x + 4 & | & x + 4 \\ -x + 3 & | & -x + 3 & | & x - 3 \end{array}$$

$$\begin{cases} x < -4 \\ -x - 4 - x + 3 = 5 \end{cases}$$

$$\begin{cases} x < -4 \\ -2x = 6 \end{cases}$$

$$\begin{cases} x < -4 \\ x = -3 \end{cases}$$

imp.

$$\begin{cases} -4 \leq x < 3 \\ x + 4 - x + 3 = 5 \end{cases}$$

$$\begin{cases} -4 \leq x < 3 \\ 0x = -2 \end{cases}$$

imp.

$$\begin{cases} x \geq 3 \\ x + 4 + x - 3 = 5 \end{cases}$$

$$\begin{cases} x \geq 3 \\ 2x = 4 \end{cases}$$

$$\begin{cases} x \geq 3 \\ x = 2 \end{cases}$$

imp.

imp.

6. $|3x + 7| < 5$

$$\begin{cases} 3x + 7 < 5 \\ 3x + 7 > -5 \end{cases}$$

$$\begin{cases} 3x < -2 \\ 3x > -12 \end{cases}$$

$$\begin{cases} x < -\frac{2}{3} \\ x > -4 \end{cases}$$

$$-4 < x < -\frac{2}{3}$$

7. $|7x + 1| > 4$

$$7x + 1 < -4 \quad \vee \quad 7x + 1 > 4$$

$$7x < -5 \quad \vee \quad 7x > 3$$

$$x < -\frac{5}{7} \quad \vee \quad x > \frac{3}{7}$$