A

1. $\dfrac{7+2\sqrt{6}}{\sqrt{2}} \cdot \dfrac{4}{\sqrt{2}+2\sqrt{3}}$

$= \dfrac{4\,(7+2\sqrt{6})}{2+2\sqrt{6}} = \dfrac{4\,(7+2\sqrt{6})}{2\,(1+\sqrt{6})} \cdot \dfrac{\sqrt{6}-1}{\sqrt{6}-1} =$

$= \dfrac{2\,(7\sqrt{6}-7+12-2\sqrt{6})}{5} = \dfrac{10\,(\sqrt{6}+1)}{5} = 2\,(\sqrt{6}+1)$

2. $\left(\sqrt{2\sqrt{5}}\right)^2 + (2+\sqrt{5})^2 + (3-\sqrt{5})^2 + (\sqrt{7}-7\sqrt{2})\,(\sqrt{7}+7\sqrt{2})$

$= 2\sqrt{5} + 4 + 4\sqrt{5} + 5 + 9 - 6\sqrt{5} + 5 + 7 - 98 = -68$

3. $\dfrac{3\sqrt{8}+\sqrt{125}-2\sqrt{18}+4\sqrt{50}-\sqrt{45}}{\sqrt{5}+10\sqrt{2}}$

$= \dfrac{6\sqrt{2}+5\sqrt{5}-6\sqrt{2}+20\sqrt{2}-3\sqrt{5}}{\sqrt{5}+10\sqrt{2}} = \dfrac{20\sqrt{2}+2\sqrt{5}}{\sqrt{5}+10\sqrt{2}} = \dfrac{2\,(10\sqrt{2}+\sqrt{5})}{\sqrt{5}+10\sqrt{2}} = 2$

4. $\left(5^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} : 5^{-\frac{3}{4}} = 5^{\frac{1}{4}} \cdot 5^{\frac{1}{2}} : 5^{-\frac{3}{4}} = 5^{\frac{1}{4}+\frac{1}{2}+\frac{3}{4}} = 5^{1+\frac{1}{2}} = 5^{\frac{3}{2}}$

5. $\left\{\left[\left(x \cdot x^{\frac{1}{3}}\right)^{\frac{1}{3}} \cdot \left(x^{-2}\, x^{\frac{5}{3}}\right)^{\frac{1}{3}}\right]^6\right\}^{-\frac{1}{2}} \cdot \left[(x^3\, y)^{-\frac{2}{3}}\right]^{-\frac{3}{4}}$

$= \left\{\left[\left(x^{\frac{4}{3}}\right)^{\frac{1}{3}} \cdot \left(x^{-\frac{1}{3}}\right)^{\frac{1}{3}}\right]^6\right\}^{-\frac{1}{2}} \cdot \left[x^{-2}\, y^{-\frac{2}{3}}\right]^{-\frac{3}{4}} = \left\{\left[x^{\frac{4}{9}} \cdot x^{-\frac{1}{9}}\right]^6\right\}^{-\frac{1}{2}} \cdot x^{\frac{3}{2}}\, y^{\frac{1}{2}} =$

$= \left\{\left[x^{\frac{1}{3}}\right]^6\right\}^{-\frac{1}{2}} \cdot x^{\frac{3}{2}}\, y^{\frac{1}{2}} = x^{-1} \cdot x^{\frac{3}{2}}\, y^{\frac{1}{2}} = x^{\frac{1}{2}}\, y^{\frac{1}{2}}$

6. $(x-\sqrt{3})\,(x+\sqrt{6}) + 3\sqrt{2} = x\,(x+\sqrt{6}) - 3$

$x^2 - x\sqrt{3} + x\sqrt{6} - 3\sqrt{2} + 3\sqrt{2} = x^2 + x\sqrt{6} - 3$

$x\sqrt{3} = 3$

$3x = 3\sqrt{3}$ $\qquad x = \sqrt{3}$