A

1. Razionalizza i denominatori delle seguenti frazioni:

$$\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$$

$$\frac{\sqrt{a} - a}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{a - a\sqrt{a}}{a} = \frac{a\,(1 - \sqrt{a})}{a} = 1 - \sqrt{a}$$

$$\frac{3}{\sqrt{7}+1} \cdot \frac{\sqrt{7}-1}{\sqrt{7}-1} = \frac{3\,(\sqrt{7}-1)}{7-1} = \frac{\sqrt{7}-1}{2}$$

$$\frac{22\sqrt{3}}{3\sqrt{3}+4} \cdot \frac{3\sqrt{3}-4}{3\sqrt{3}-4} = \frac{22\,(9 - 4\sqrt{3})}{27-16} = 18 - 8\sqrt{3}$$

2. Poni sotto forma di un unico radicale i seguenti radicali:

$$\sqrt{5\sqrt[3]{\frac{3}{50}}} = \sqrt{\sqrt[3]{5^3 \cdot \frac{3}{50}}} = \sqrt[6]{5^3 \cdot \frac{3}{2 \cdot 5^2}} = \sqrt[6]{\frac{15}{2}}$$

$$\sqrt[3]{\frac{1}{4}\sqrt{4\sqrt{2}}} = \sqrt[3]{\frac{1}{4}\sqrt{\sqrt{4^2 \cdot 2}}} = \sqrt[3]{\frac{1}{4}\sqrt[4]{2^4 \cdot 2}} = \sqrt[3]{\frac{1}{2^2}\sqrt[4]{2^5}} = \sqrt[3]{\sqrt[4]{\frac{1}{2^8} \cdot 2^5}} = \sqrt[12]{\frac{1}{2^3}} = \sqrt[4]{\frac{1}{2}}$$

3. Trasporta fuori dal segno di radice tutti i fattori possibili nei seguenti radicali:

$$\sqrt{a^6 b} = a^3\sqrt{b}$$

$$\sqrt[3]{125\,a^3 b} = \sqrt[3]{5^3\,a^3 b} = 5a\sqrt[3]{b}$$

$$\sqrt[3]{8a^3 b^9 c^2} = \sqrt[3]{2^3 a^3 b^9 c^2} = 2ab^3\sqrt[3]{c^2}$$

$$\sqrt{16\,b^4 c} = \sqrt{2^4\,b^4 c} = 2^2 b^2\sqrt{c}$$

4. Semplifica le seguenti espressioni:

a. $$\frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}+2} - \sqrt{7} =$$

$$= \frac{1}{\sqrt{7}+\sqrt{6}} \cdot \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{5}} \cdot \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}+2} \cdot \frac{\sqrt{5}-2}{\sqrt{5}-2} - \sqrt{7} =$$

$$= \sqrt{7} - \sqrt{6} + \sqrt{6} - \sqrt{5} + \sqrt{5} - 2 - \sqrt{7} = -2$$

b. $(4+\sqrt{2})^2-(2\sqrt{2}-1)^2-3(4\sqrt{2}+2)=$

$=16+2+8\sqrt{2}-(8-4\sqrt{2}+1)-12\sqrt{2}-6=$

$=16+2+8\sqrt{2}-8+4\sqrt{2}-1-12\sqrt{2}-6=$ 3

c. $(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})+(\sqrt{3}-2)^2+\sqrt{48}=$

$=2-3+3+4-4\sqrt{3}+\sqrt{2^4\cdot 3}=$

$=6-4\sqrt{3}+4\sqrt{3}=$ 6

d. $\sqrt{x\sqrt[7]{\frac{1}{x}}}\cdot\sqrt[5]{x^3\sqrt{\frac{1}{x^5}}}\cdot\sqrt{\frac{1}{x}}=$

$=\sqrt{\sqrt[7]{x^7\cdot\frac{1}{x}}}\cdot\sqrt[5]{\sqrt{x^6\cdot\frac{1}{x^5}}}\cdot\sqrt{\frac{1}{x}}=$

$=\sqrt[14]{x^6}\cdot\sqrt[10]{x}\cdot\sqrt{\frac{1}{x}}=\sqrt[7]{x^3}\cdot\sqrt[10]{x}\cdot\sqrt{\frac{1}{x}}=$

$=\sqrt[70]{x^{30}\cdot x^7\cdot\frac{1}{x^{35}}}=\sqrt[70]{x^2}=$ $\sqrt[35]{x}$

e. $(\sqrt[4]{a})^2\cdot(\sqrt[3]{a^2})^3\cdot\sqrt{a}=$

$=\sqrt{a}\cdot a^2\cdot\sqrt{a}=a\cdot a^2=$ a^3