1. $(2x-1)^3 + 9x(x-5) \geq x^2(8x-3) - 40x$

$8x^3 - 12x^2 + 6x - 1 + 9x^2 - 45x \geq 8x^3 - 3x^2 - 40x$

$x \geq 1$

2. $\frac{7x}{2x+4} + \frac{5}{x-2} \geq \frac{6x-16-7x^2}{8-2x^2}$

$\frac{7x}{2(x+2)} + \frac{5}{x-2} \geq -\frac{6x-16-7x^2}{2(x-2)(x+2)}$

$\frac{7x^2 - 14x + 10x + 20 + 6x - 16 - 7x^2}{2(x-2)(x+2)} \geq 0$

$\frac{2(x+2)}{2(x-2)(x+2)} \geq 0 \qquad x \neq -2$

$\frac{1}{x-2} \geq 0$

$x > 2$

3. $(x^4 - 16)(9x^2 - 7x - 2)(x+3) \geq 0$

$(x^2+4)(x+2)(x-2)(9x+2)(x-1)(x+3) \geq 0$

$-3 \leq x \leq -2 \vee -\frac{2}{9} \leq x \leq 1 \vee x \geq 2$

4. $\begin{cases} x^2 - 4 \leq 0 \\ 9 + 3x^2 \geq 3(x-1)(x+1) - 6(1-x) \\ \frac{2x+1}{x-5} \leq 0 \end{cases}$

$\begin{cases} x \leq -2 \vee x \geq 2 \\ 9 + 3x^2 \geq 3x^2 - 3 - 6 + 6x \\ -\frac{1}{2} \leq x < 5 \end{cases}$

$\begin{cases} x \leq -2 \vee x \geq 2 \\ 18 \geq 6x \\ -\frac{1}{2} \leq x < 5 \end{cases} \qquad \begin{cases} x \leq -2 \vee x \geq 2 \\ x \leq 3 \\ -\frac{1}{2} \leq x < 5 \end{cases}$

$-\frac{1}{2} \leq x \leq 2$

5. $|2x-1|-|x+4|=x+5$

$\begin{cases} x<-4 \\ -2x+1+x+4=x+5 \end{cases}$ $\begin{cases} x<-4 \\ x=0 \end{cases}$ *imp.*

$\begin{cases} -4\le x<\frac{1}{2} \\ -2x+1-x-4=x+5 \end{cases}$ $\begin{cases} -4\le x<\frac{1}{2} \\ -4x=8 \end{cases}$ $x=-2$

$\begin{cases} x\ge\frac{1}{2} \\ 2x-1-x-4=x+5 \end{cases}$ $\begin{cases} x\ge\frac{1}{2} \\ 0=10 \end{cases}$ *imp.*

6. $|5x+1|\ge 4$

$5x+1\le -4 \quad \vee \quad 5x+1\ge 4$

$5x\le -5 \quad \vee \quad 5x\ge 3$ $x\le -1 \ \vee \ x\ge\frac{3}{5}$

7. $\left|2x+\frac{2}{3}\right|\le x-\frac{2}{3}$

$\begin{cases} x\ge -\frac{1}{3} \\ 2x+\frac{2}{3}\le x-\frac{2}{3} \end{cases}$ $\begin{cases} x\ge -\frac{1}{3} \\ x\le --\frac{4}{3} \end{cases}$ *imp.*

$\begin{cases} x<-\frac{1}{3} \\ -2x-\frac{2}{3}\le x-\frac{2}{3} \end{cases}$ $\begin{cases} x<-\frac{1}{3} \\ x\ge 0 \end{cases}$ *imp.*

imp.

8. $\left|\left|x-1\right|-\left|7x+5\right|\right|+\left|3x+8\right|\le -15$ *imp.*