1. Razionalizza i denominatori delle seguenti frazioni:

$$\frac{6}{\sqrt{8}} = \frac{6}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{2}}$$

$$\frac{\sqrt{a}}{a - b\sqrt{a}} \cdot \frac{a + b\sqrt{a}}{a + b\sqrt{a}} = \frac{a\sqrt{a} + ab}{a^2 - ab^2} = \frac{a(\sqrt{a} + b)}{a(a - b)} = \boxed{\frac{\sqrt{a} + b}{a - b}}$$

$$\frac{5}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{5(2 + \sqrt{3})}{4 - 3} = \boxed{5(2 + \sqrt{3})}$$

$$\frac{30}{5\sqrt{3} - 3\sqrt{5}} \cdot \frac{5\sqrt{3} + 3\sqrt{5}}{5\sqrt{3} + 3\sqrt{5}} = \frac{30(5\sqrt{3} + 3\sqrt{5})}{75 - 45} = \boxed{5\sqrt{3} + 3\sqrt{5}}$$

2. Poni sotto forma di un unico radicale i seguenti radicali:

$$\sqrt[3]{4\sqrt{\frac{1}{2}}} = \sqrt[3]{2^2\sqrt{\frac{1}{2}}} = \sqrt[3]{\sqrt{2^4 \cdot \frac{1}{2}}} = \sqrt[6]{2^3} = \boxed{\sqrt{2}}$$

$$\sqrt[5]{8\sqrt{\frac{1}{32}}} = \sqrt[5]{2^3\sqrt{\frac{1}{2^5}}} = \sqrt[5]{\sqrt{2^6 \cdot \frac{1}{2^5}}} = \boxed{\sqrt[10]{2}}$$

3. Trasforma i seguenti radicali doppi nella somma di due radicali semplici:

$$\sqrt{6 + 2\sqrt{5}} = \sqrt{(\sqrt{5} + 1)^2} = \boxed{\sqrt{5} + 1}$$

$$\sqrt{6 - \sqrt{11}} = \sqrt{\frac{1}{2}}\sqrt{12 - 2\sqrt{11}} = \frac{\sqrt{(\sqrt{11} - 1)^2}}{\sqrt{2}} = \frac{\sqrt{11} - 1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{22} - \sqrt{2}}{2}}$$

$$\sqrt{6 - \sqrt{32}} = \sqrt{6 - 2\sqrt{8}} = \sqrt{(\sqrt{4} - \sqrt{2})^2} = \boxed{2 - \sqrt{2}}$$

4. Semplifica le seguenti espressioni:

a. $\sqrt[4]{162} - \sqrt[4]{32} + 5\sqrt[3]{16} - \sqrt[3]{54} + \sqrt[3]{250} - \sqrt[4]{2} =$

$= 3\sqrt[4]{2} - 2\sqrt[4]{2} + 10\sqrt[3]{2} - 3\sqrt[3]{2} + 5\sqrt[3]{2} - \sqrt[4]{2} = 12\sqrt[3]{2}$

b. $[(2\sqrt{5}+1)(2\sqrt{5}-1) - (\sqrt{5}-1)^2 - (\sqrt{5}-4)^2 - 10\sqrt{5}] : 2 =$

$= [20 - 1 - (5 - 2\sqrt{5} + 1) - (5 - 8\sqrt{5} + 16) - 10\sqrt{5}] : 2 =$

$= [20 - 1 - 5 + 2\sqrt{5} - 1 - 5 + 8\sqrt{5} - 16 - 10\sqrt{5}] : 2 = -8 : 2 = -4$

c. $\dfrac{3 + 2\sqrt{2}}{2 + \sqrt{2}} \cdot \dfrac{\sqrt{2} - 1}{\sqrt{2}} =$

$= \dfrac{3\sqrt{2} - 3 + 4 - 2\sqrt{2}}{2\sqrt{2} + 2} = \dfrac{\sqrt{2} + 1}{2(\sqrt{2} + 1)} = \dfrac{1}{2}$

d. $(\sqrt{6} - 1)\sqrt{7 + 2\sqrt{6}} - \sqrt{6 - \sqrt{11}} \cdot \sqrt{6 + \sqrt{11}} =$

$= (\sqrt{6} - 1)\sqrt{(\sqrt{6} + 1)^2} - \sqrt{36 - 11} =$

$= (\sqrt{6} - 1)(\sqrt{6} + 1) - \sqrt{25} = 6 - 1 - 5 = 0$

e. $\dfrac{1}{\sqrt{a} - \sqrt{b}} + \dfrac{3\sqrt{a} - \sqrt{b}}{a - b} =$

$= \dfrac{1}{\sqrt{a} - \sqrt{b}} \cdot \dfrac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} + \dfrac{3\sqrt{a} - \sqrt{b}}{a - b} =$

$= \dfrac{\sqrt{a} + \sqrt{b}}{a - b} + \dfrac{3\sqrt{a} - \sqrt{b}}{a - b} = \dfrac{\sqrt{a} + \sqrt{b} + 3\sqrt{a} - \sqrt{b}}{a - b} = \dfrac{4\sqrt{a}}{a - b}$