

Semplifica le seguenti espressioni:

$$\begin{aligned}
 1. \quad & (x + 2y)^2 - (x - 2y)^2 - 8xy \\
 &= x^2 + 4xy + 4y^2 - (x^2 - 4xy + 4y^2) - 8xy = \\
 &= x^2 + 4xy + 4y^2 - x^2 + 4xy - 4y^2 - 8xy = \mathbf{0}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \left(x^2y - \frac{1}{2}xy^2\right)\left(x^2y + \frac{1}{2}xy^2\right) + \left(\frac{1}{2}xy^2 + \frac{1}{4}x^3\right)^2 - \frac{1}{4}x^4\left(5y^2 + \frac{1}{4}x^2\right) \\
 &= x^4y^2 - \frac{1}{4}x^2y^4 + \frac{1}{4}x^2y^4 + \frac{1}{4}x^4y^2 + \frac{1}{16}x^6 - \frac{5}{4}x^4y^2 - \frac{1}{16}x^6 = \mathbf{0}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & (5x - 1)(5x + 1) - 9x(x - 3) - (4x + 3)^2 - 3x \\
 &= 25x^2 - 1 - 9x^2 + 27x - (16x^2 + 24x + 9) - 3x = \\
 &= 25x^2 - 1 - 9x^2 + 27x - 16x^2 - 24x - 9 - 3x = \mathbf{-10}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & (2x - 3)(8x + 1) + (3x - 1)^2 - (5x - 2)^2 + 2(4x + 3) \\
 &= 16x^2 + 2x - 24x - 3 + 9x^2 - 6x + 1 - (25x^2 - 20x + 4) + 8x + 6 = \\
 &= 16x^2 + 2x - 24x - 3 + 9x^2 - 6x + 1 - 25x^2 + 20x - 4 + 8x + 6 = \mathbf{0}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & (a - 2)^3 - (a + 2)^2 + a - 2 - (a + 2)^3 - a(2 - 13a) + 22 \\
 &= a^3 - 6a^2 + 12a - 8 - (a^2 + 4a + 4) + a - 2 - (a^3 + 6a^2 + 12a + 8) - 2a + 13a^2 + 22 = \\
 &= a^3 - 6a^2 + 12a - 8 - a^2 - 4a - 4 + a - 2 - a^3 - 6a^2 - 12a - 8 - 2a + 13a^2 + 22 = \mathbf{-5a}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \left[\left(\frac{1}{2}x^2y - x\right)^2 - x^2\right]^2 - \frac{1}{64}x^9y^5 - \left(\frac{x^3y}{4}\right)^2 \cdot \left(1 - \frac{1}{4}xy\right)(x^2y^2 + 32) \\
 &= \left[\frac{1}{4}x^4y^2 - x^3y + x^2 - x^2\right]^2 - \frac{1}{64}x^9y^5 - \frac{x^6y^2}{16} \cdot \left(x^2y^2 + 32 - \frac{1}{4}x^3y^3 - 8xy\right) = \\
 &= \left[\frac{1}{4}x^4y^2 - x^3y\right]^2 - \frac{1}{64}x^9y^5 - \frac{1}{16}x^8y^4 - 2x^6y^2 + \frac{1}{64}x^9y^5 + \frac{1}{2}x^7y^3 = \\
 &= \frac{1}{16}x^8y^4 + x^6y^2 - \frac{1}{2}x^7y^3 - \frac{1}{16}x^8y^4 - 2x^6y^2 + \frac{1}{2}x^7y^3 = \mathbf{-x^6y^2}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & 3x [4x(x-7) - (2x+1)(x-3) - (x-4)^2] - 15x^2 \left(\frac{1}{5}x - 3 \right) \\
 &= 3x [4x^2 - 28x - (2x^2 - 6x + x - 3) - (x^2 - 8x + 16)] - 3x^3 + 45x^2 = \\
 &= 3x [4x^2 - 28x - 2x^2 + 6x - x + 3 - x^2 + 8x - 16] - 3x^3 + 45x^2 = \\
 &= 3x [x^2 - 15x - 13] - 3x^3 + 45x^2 = 3x^3 - 45x^2 - 39x - 3x^3 + 45x^2 = \boxed{-39x}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \left\{ -\left[\frac{4}{9}a - (2a + 3b) - \frac{1}{9}a + 1 \right] \right\}^2 - \left(\frac{5}{3}a + 1 + 3b \right)^2 + 4 \left(\frac{5}{3}a + 3b \right) \\
 &= \left\{ -\left[\frac{4}{9}a - 2a - 3b - \frac{1}{9}a + 1 \right] \right\}^2 - \left(\frac{5}{3}a + 1 + 3b \right)^2 + \frac{20}{3}a + 12b = \\
 &= \left\{ -\left[-\frac{5}{3}a - 3b + 1 \right] \right\}^2 - \left(\frac{25}{9}a^2 + 1 + 9b^2 + \frac{10}{3}a + 10ab + 6b \right) + \frac{20}{3}a + 12b = \\
 &= \left\{ \frac{5}{3}a + 3b - 1 \right\}^2 - \frac{25}{9}a^2 - 1 - 9b^2 - \frac{10}{3}a - 10ab - 6b + \frac{20}{3}a + 12b = \\
 &= \frac{25}{9}a^2 + 1 + 9b^2 - \frac{10}{3}a + 10ab - 6b - \frac{25}{9}a^2 - 1 - 9b^2 - 10ab + \frac{10}{3}a + 6b = \boxed{0}
 \end{aligned}$$

Trova quoziente e resto della seguente divisione:

$$9. \quad (x^7 - x^6 - 3x^5 + 3x^4 + x) : (x - 1)$$

$$Q(x) = x^6 - 3x^4 + 1$$

$$R(x) = 1$$

$$\begin{array}{r|rrrrrrr}
 1 & 1 & -1 & -3 & 3 & 0 & 0 & 1 & 0 \\
 & & 1 & 0 & -3 & 0 & 0 & 0 & 1 \\
 \hline
 & 1 & 0 & -3 & 0 & 0 & 0 & 1 & 1
 \end{array}$$