

Semplifica le seguenti espressioni:

$$\begin{aligned}
 1. \quad & \left[ \left( \frac{1}{2} a - 1 \right)^2 - \frac{1}{4} a^2 \right]^2 (1+a)^2 - \left[ a^2 - \left( \frac{1}{2} - a \right)^2 - \frac{7}{4} \right]^2 (a+2)^2 \\
 &= \left[ \frac{1}{4} a^2 - a + 1 - \frac{1}{4} a^2 \right]^2 (1+a)^2 - \left[ a^2 - \left( \frac{1}{4} - a + a^2 \right) - \frac{7}{4} \right]^2 (a+2)^2 = \\
 &= (1-a)^2 (1+a)^2 - \left[ a^2 - \frac{1}{4} + a - a^2 - \frac{7}{4} \right]^2 (a+2)^2 = \\
 &= (1-a^2)^2 - (a-2)^2 (a+2)^2 = 1 - 2a^2 + a^4 - (a^2 - 4)^2 = \\
 &= 1 - 2a^2 + a^4 - (a^4 - 8a^2 + 16) = 1 - 2a^2 + a^4 - a^4 + 8a^2 - 16 = \mathbf{6a^2 - 15}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 8(a-1)^3 + 4(a-1)^2 + (a^2 - 4a + 2)^2 - a^2(a^2 - 1) \\
 &= 8(a^3 - 3a^2 + 3a - 1) + 4(a^2 - 2a + 1) + a^4 + 16a^2 + 4 - 8a^3 + 4a^2 - 16a - a^4 + a^2 = \\
 &= 8a^3 - 24a^2 + 24a - 8 + 4a^2 - 8a + 4 + a^4 + 16a^2 + 4 - 8a^3 + 4a^2 - 16a - a^4 + a^2 = \mathbf{a^2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & (2x+3)^3 - (2x-3)^3 - 6[(2x-3)^2 + 4x(2x+3)] \\
 &= 8x^3 + 36x^2 + 54x + 27 - (8x^3 - 36x^2 + 54x - 27) - 6(4x^2 - 12x + 9 + 8x^2 + 12x) = \\
 &= 8x^3 + 36x^2 + 54x + 27 - 8x^3 + 36x^2 - 54x + 27 - 24x^2 + 72x - 54 - 48x^2 - 72x = \mathbf{0}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & (3a-b)^2 + (3a+4b)^2 - 2(3a+2b)(3a-2b) - (-5b)^2 \\
 &= 9a^2 - 6ab + b^2 + 9a^2 + 24ab + 16b^2 - 2(9a^2 - 4b^2) - (25b^2) = \\
 &= 9a^2 - 6ab + b^2 + 9a^2 + 24ab + 16b^2 - 18a^2 + 8b^2 - 25b^2 = \mathbf{18ab}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{3}{2} x^2 y^2 + \left[ x^2 - \left( \frac{x}{2} - y \right) \left( -\frac{x}{2} - y \right) \right]^2 - \frac{25}{16} x^4 + (2xy - y^2)(2xy + y^2) \\
 &= \frac{3}{2} x^2 y^2 + \left[ x^2 - \left( -\frac{x^2}{4} + y^2 \right) \right]^2 - \frac{25}{16} x^4 + 4x^2 y^2 - y^4 = \\
 &= \frac{3}{2} x^2 y^2 + \left[ x^2 + \frac{x^2}{4} - y^2 \right]^2 - \frac{25}{16} x^4 + 4x^2 y^2 - y^4 = \\
 &= \frac{3}{2} x^2 y^2 + \left[ \frac{5}{4} x^2 - y^2 \right]^2 - \frac{25}{16} x^4 + 4x^2 y^2 - y^4 = \\
 &= \frac{3}{2} x^2 y^2 + \frac{25}{16} x^4 - \frac{5}{2} x^2 y^2 + y^4 - \frac{25}{16} x^4 + 4x^2 y^2 - y^4 = \mathbf{3x^2 y^2}
 \end{aligned}$$

Verifica le seguenti identità:

$$6. \quad 2(x + y)^2 - (x + 2y)^2 = (x + y)(x - y) - y^2$$

$$2(x^2 + 2xy + y^2) - (x^2 + 4xy + 4y^2) = x^2 - y^2 - y^2$$

$$2x^2 + 4xy + 2y^2 - x^2 - 4xy - 4y^2 = x^2 - 2y^2$$

$$x^2 - 2y^2 = x^2 - 2y^2$$

$$7. \quad (2x - 1)^3 - 3x(2x - 1)(x - 1) = x^3 + (x - 1)^3$$

$$8x^3 - 12x^2 + 6x - 1 - 3x(2x^2 - 2x - x + 1) = x^3 + x^3 - 3x^2 + 3x - 1$$

$$8x^3 - 12x^2 + 6x - 1 - 6x^3 + 6x^2 + 3x^2 - 3x = 2x^3 - 3x^2 + 3x - 1$$

$$2x^3 - 3x^2 + 3x - 1 = 2x^3 - 3x^2 + 3x - 1$$

Trova quoziente e resto della seguente divisione:

$$8. \quad (9a^3 + 4a^2 - 5a + 1) : (a + 3)$$

$$Q(x) = 9a^2 - 23a + 64$$

$$R(x) = -191$$

$$\begin{array}{r|rrr|r} -3 & 9 & 4 & -5 & 1 \\ & & -27 & 69 & -192 \\ \hline & 9 & -23 & 64 & -191 \end{array}$$