

1. $-(xy + 1)(xy - 1) + (xy + 1)^2 - 2xy - 1$

$$= -(x^2y^2 - 1) + x^2y^2 + 1 + 2xy - 2xy - 1 =$$

$$= -x^2y^2 + 1 + x^2y^2 + 1 + 2xy - 2xy - 1 = \mathbf{1}$$

2. $(2a + 1)^3 + (2a - 1)^3 - 4a(4a^2 + 3) + 2$

$$= 8a^3 + 12a^2 + 6a + 1 + 8a^3 - 12a^2 + 6a - 1 - 16a^3 - 12a + 2 = \mathbf{2}$$

3. $(3x - y^2)^2 - (3x + y^2)(3x - 2y^2) - y^2(y^2 - 3x + 2y^2) + 3$

$$= 9x^2 - 6xy^2 + y^4 - (9x^2 - 6xy^2 + 3xy^2 - 2y^4) - y^4 + 3xy^2 - 2y^4 + 3 =$$

$$= 9x^2 - 6xy^2 + y^4 - 9x^2 + 6xy^2 - 3xy^2 + 2y^4 - y^4 + 3xy^2 - 2y^4 + 3 = \mathbf{3}$$

4. $(5ab - 3a)^2 - 2(5ab - 3a)(3a + 5ab) + (4a + 5ab)^2 - 43a^2 - 10a^2b + 4$

$$= 25a^2b^2 - 30a^2b + 9a^2 - 2(25a^2b^2 - 9a^2) + 16a^2 + 40a^2b + 25a^2b^2 - 43a^2 - 10a^2b + 4 =$$

$$= 25a^2b^2 - 30a^2b + 9a^2 - 50a^2b^2 + 18a^2 + 16a^2 + 40a^2b + 25a^2b^2 - 43a^2 - 10a^2b + 4 = \mathbf{4}$$

5. $[-2a(-a^2b)^2] + (-2a)^3(-ab)^2 - (-a)^5(-4b)^2 + (-3a)^0 - 6a^5b^2 + 4$

$$= [-2a(a^4b^2)] + (-8a^3)(a^2b^2) - (-a^5)(16b^2) + 1 - 6a^5b^2 + 4 =$$

$$= -2a^5b^2 - 8a^5b^2 + 16a^5b^2 + 1 - 6a^5b^2 + 4 = \mathbf{5}$$

6. $[(a - 1)(a + 1)(a^2 + 1)]^2 - [(2a - 1)(2a + 1)(4a^2 + 1)]^2 + 255a^8 - 30a^4 + 6$

$$= [(a^2 - 1)(a^2 + 1)]^2 - [(4a^2 - 1)(4a^2 + 1)]^2 + 255a^8 - 30a^4 + 6 =$$

$$= (a^4 - 1)^2 - (16a^4 - 1)^2 + 255a^8 - 30a^4 + 6 =$$

$$= a^8 - 2a^4 + 1 - (256a^8 - 32a^4 + 1) + 255a^8 - 30a^4 + 6 =$$

$$= a^8 - 2a^4 + 1 - 256a^8 + 32a^4 - 1 + 255a^8 - 30a^4 + 6 = \mathbf{6}$$

$$\begin{aligned}
 7. \quad & a(a-2b)^3 - [(a+2b)^2 - (2a+b)^2]^2 + 2ab(3a^2 + 4b^2 - 15ab) + 8a^4 + 9b^4 + 7 \\
 &= a(a^3 - 6a^2b + 12ab^2 - 8b^3) - [a^2 + 4ab + 4b^2 - (4a^2 + 4ab + b^2)]^2 + 6a^3b + 8ab^3 + \\
 &\quad - 30a^2b^2 + 8a^4 + 9b^4 + 7 = \\
 &= a^4 - 6a^3b + 12a^2b^2 - 8ab^3 - (a^2 + 4ab + 4b^2 - 4a^2 - 4ab - b^2)^2 + 6a^3b + 8ab^3 + \\
 &\quad - 30a^2b^2 + 8a^4 + 9b^4 + 7 = \\
 &= a^4 - 6a^3b + 12a^2b^2 - 8ab^3 - (3b^2 - 3a^2)^2 + 6a^3b + 8ab^3 - 30a^2b^2 + 8a^4 + 9b^4 + 7 = \\
 &= a^4 - 6a^3b + 12a^2b^2 - 8ab^3 - (9b^4 - 18a^2b^2 + 9a^4) + 6a^3b + 8ab^3 - 30a^2b^2 + 8a^4 + 9b^4 + 7 = \\
 &= a^4 - 6a^3b + 12a^2b^2 - 8ab^3 - 9b^4 + 18a^2b^2 - 9a^4 + 6a^3b + 8ab^3 - 30a^2b^2 + 8a^4 + 9b^4 + 7 = 7
 \end{aligned}$$

Esegui la seguente divisione applicando la regola di Ruffini:

$$(4x^3 + 7x^2 + 3x) : (x + 1)$$

$$\begin{array}{r|rrr|r}
 -1 & 4 & 7 & 3 & 0 \\
 & & -4 & -3 & 0 \\
 \hline
 & 4 & 3 & 0 & 0
 \end{array}$$

$$Q(x) = 4x^2 + 3x$$

$$R(x) = 0$$