B

Verifica le seguenti identità:

1. $2\ sen\ \alpha \cos\alpha = (\cos\alpha + sen\ \alpha)^2 - 1$

$2\ sen\ \alpha \cos\alpha = sen^2\ \alpha + \cos^2\alpha + 2\ sen\ \alpha \cos\alpha - 1$

$2\ sen\ \alpha \cos\alpha = 1 + 2\ sen\ \alpha \cos\alpha - 1$

$2\ sen\ \alpha \cos\alpha = 2\ sen\ \alpha \cos\alpha$

2. $(tg\ \alpha - sen\ \alpha)\ ctg\ \alpha = 1 - \cos\alpha$

$tg\ \alpha \cdot ctg\ \alpha - sen\ \alpha \cdot ctg\ \alpha = 1 - \cos\alpha$

$1 - sen\ \alpha \cdot \dfrac{\cos\alpha}{sen\ \alpha} = 1 - \cos\alpha$

$1 - \cos\alpha = 1 - \cos\alpha$

3. $(1 + ctg^2\ \alpha)(1 - \cos^2\alpha) = 1$

$\left(1 + \dfrac{\cos^2\alpha}{sen^2\ \alpha}\right)(sen^2\ \alpha) = 1$

$\dfrac{sen^2\ \alpha + \cos^2\alpha}{sen^2\ \alpha} \cdot sen^2\ \alpha = 1$

$sen^2\ \alpha + \cos^2\alpha = 1$

$1 = 1$

4. $sen^2\ \alpha\ ctg^2\ \alpha + \cos^2\alpha\ tg^2\ \alpha = 1$

$sen^2\ \alpha \dfrac{\cos^2\alpha}{sen^2\ \alpha} + \cos^2\alpha \dfrac{sen^2\ \alpha}{\cos^2\alpha} = 1$

$\cos^2\alpha + sen^2\ \alpha = 1$

$1 = 1$

5. $(ctg\ \alpha + tg\ \alpha)\cos\alpha = \dfrac{1}{sen\ \alpha}$

$\dfrac{\cos\alpha}{sen\ \alpha} \cdot \cos\alpha + \dfrac{sen\ \alpha}{\cos\alpha} \cdot \cos\alpha = \dfrac{1}{sen\ \alpha}$

$\dfrac{\cos^2\alpha}{sen\ \alpha} + sen\ \alpha = \dfrac{1}{sen\ \alpha}$

$\dfrac{\cos^2\alpha + sen^2\ \alpha}{sen\ \alpha} = \dfrac{1}{sen\ \alpha}$

$\dfrac{1}{sen\ \alpha} = \dfrac{1}{sen\ \alpha}$

6. $\dfrac{1}{2 - \cos^2\alpha} = \dfrac{1 + ctg^2\ \alpha}{2 + ctg^2\ \alpha}$

$\dfrac{1}{2 - \dfrac{ctg^2\ \alpha}{1 + ctg^2\ \alpha}} = \dfrac{1 + ctg^2\ \alpha}{2 + ctg^2\ \alpha}$

$\dfrac{1}{\dfrac{2 + 2\ ctg^2\ \alpha - ctg^2\ \alpha}{1 + ctg^2\ \alpha}} = \dfrac{1 + ctg^2\ \alpha}{2 + ctg^2\ \alpha}$

$\dfrac{1}{\dfrac{2 + ctg^2\ \alpha}{1 + ctg^2\ \alpha}} = \dfrac{1 + ctg^2\ \alpha}{2 + ctg^2\ \alpha}$

$\dfrac{1 + ctg^2\ \alpha}{2 + ctg^2\ \alpha} = \dfrac{1 + ctg^2\ \alpha}{2 + ctg^2\ \alpha}$

7. $sen^4\ \alpha + \cos^4\ \alpha = 1 - 2\ ctg^2\ \alpha\ sen^4\ \alpha$

$$sen^4\ \alpha + \cos^4\ \alpha = 1 - 2\ \frac{\cos^2\ \alpha}{sen^2\ \alpha} \cdot sen^4\ \alpha$$

$$sen^4\ \alpha + \cos^4\ \alpha = 1 - 2\ \cos^2\ \alpha\ sen^2\ \alpha$$

$$sen^4\ \alpha + \cos^4\ \alpha + 2\ \cos^2\ \alpha\ sen^2\ \alpha = 1$$

$$(sen^2\ \alpha + \cos^2\ \alpha)^2 = 1 \qquad 1 = 1$$

8. $\cos^2\ \alpha + sen^2\ \alpha\ \cos^2\ \alpha + sen^4\ \alpha = 1$

$$\cos^2\ \alpha + sen^2\ \alpha\ (\cos^2\ \alpha + sen^2\ \alpha) = 1$$

$$\cos^2\ \alpha + sen^2\ \alpha = 1 \qquad 1 = 1$$

Semplifica le seguenti espressioni, usando le formule degli archi associati:

9. $$\frac{sen\left(\frac{\pi}{2} + \alpha\right)\cos\left(\frac{\pi}{2} - \alpha\right)}{\cos\ (\pi + \alpha)} + \frac{sen\ (\pi - \alpha)\ \cos\left(\frac{\pi}{2} + \alpha\right)}{sen\ (-\alpha)}$$

$$= \frac{\cos\ \alpha \cdot sen\ \alpha}{-\cos\ \alpha} + \frac{sen\ \alpha\ (-\ sen\ \alpha)}{-\ sen\ \alpha} = -sen\ \alpha + sen\ \alpha = 0$$

10. $$\frac{\cos\ (-\alpha) + \cos\ (\pi - \alpha) + \cos\ (\pi + \alpha)}{sen\ (-\alpha) + sen\ (\pi - \alpha) + sen\ (\pi + \alpha)}$$

$$= \frac{\cos\ \alpha - \cos\ \alpha - \cos\ \alpha}{-\ sen\ \alpha + sen\ \alpha - sen\ \alpha} = \frac{-\cos\ \alpha}{-\ sen\ \alpha} = ctg\ \alpha$$

11. $$\frac{tg\ (\pi - \alpha) - \cos\left(\frac{\pi}{2} - \alpha\right) - \cos\ (-\alpha)}{sen\ (-\alpha) + \cos\ (\pi - \alpha) - tg\ (\pi + \alpha)} = \frac{-\ tg\ \alpha - sen\ \alpha - \cos\ \alpha}{-\ sen\ \alpha - \cos\ \alpha - tg\ \alpha} = 1$$

12. $$\frac{\cos\left(\frac{\pi}{2} + \alpha\right) + sen\ (-\alpha) + sen\ (2\pi - \alpha) + \cos\left(\frac{\pi}{2} - \alpha\right)}{sen\left(\frac{\pi}{2} - \alpha\right) + \cos\ (-\alpha)}$$

$$= \frac{-\ sen\ \alpha - sen\ \alpha - sen\ \alpha + sen\ \alpha}{\cos\ \alpha + \cos\ \alpha} = \frac{-sen\ \alpha - sen\ \alpha}{2\ \cos\ \alpha} = \frac{-2\ sen\ \alpha}{2\ \cos\ \alpha} = -\ tg\ \alpha$$

Calcola il valore delle seguenti espressioni:

13. $\sqrt{3}\ sen\ \frac{\pi}{6} - 3\ sen\ \frac{4}{3}\pi + 2\ ctg\ \frac{5}{6}\pi + \sqrt{2}\cos\frac{7}{4}\pi$

$$= \sqrt{3}\cdot\frac{1}{2} - 3\left(-\frac{\sqrt{3}}{2}\right) + 2\,(-\sqrt{3}) + \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} - 2\sqrt{3} + 1 = \boxed{1}$$

14. $ctg^2\ \frac{2}{3}\pi + \cos\frac{3}{4}\pi \cdot sen\ \frac{11}{4}\pi - \frac{1}{3}\ sen\ \frac{7}{6}\pi$

$$= \left(-\frac{\sqrt{3}}{3}\right)^2 - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{3}\left(-\frac{1}{2}\right) = \frac{1}{3} - \frac{1}{2} + \frac{1}{6} = \frac{2-3+1}{6} = \boxed{0}$$

15. $2\cos\frac{5}{3}\pi + 4\ sen\ \frac{2}{3}\pi \cdot \cos\frac{5}{6}\pi - 2\ ctg\ \frac{3}{4}\pi + tg\ \frac{5}{3}\pi$

$$= 2\cdot\frac{1}{2} + 4\cdot\frac{\sqrt{3}}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right) - 2\,(-1) - \sqrt{3} = 1 - 3 + 2 - \sqrt{3} = \boxed{-\sqrt{3}}$$

16. $tg\ \frac{27}{4}\pi - sen\ \frac{33}{6}\pi + \cos\frac{45}{4}\pi - tg\ \frac{23}{3}\pi + ctg\ \frac{29}{6}\pi + sen\ \frac{17}{4}\pi$

$$= -1 - (-1) + \left(-\frac{\sqrt{2}}{2}\right) - (-\sqrt{3}) + (-\sqrt{3}) + \frac{\sqrt{2}}{2} = -1 + 1 - \frac{\sqrt{2}}{2} + \sqrt{3} - \sqrt{3} + \frac{\sqrt{2}}{2} = \boxed{0}$$