

# A

Verifica le seguenti identità:

$$1. \quad (\operatorname{sen} \alpha + \cos \alpha)^2 - 1 = 2 \operatorname{sen} \alpha \cos \alpha$$

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha + 2 \operatorname{sen} \alpha \cos \alpha - 1 = 2 \operatorname{sen} \alpha \cos \alpha$$

$$1 + 2 \operatorname{sen} \alpha \cos \alpha - 1 = 2 \operatorname{sen} \alpha \cos \alpha$$

$$2 \operatorname{sen} \alpha \cos \alpha = 2 \operatorname{sen} \alpha \cos \alpha$$

$$2. \quad (\operatorname{ctg} \alpha - \cos \alpha) \operatorname{tg} \alpha = 1 - \operatorname{sen} \alpha$$

$$\operatorname{ctg} \alpha \cdot \operatorname{tg} \alpha - \cos \alpha \cdot \operatorname{tg} \alpha = 1 - \operatorname{sen} \alpha$$

$$1 - \cos \alpha \cdot \frac{\operatorname{sen} \alpha}{\cos \alpha} = 1 - \operatorname{sen} \alpha$$

$$1 - \operatorname{sen} \alpha = 1 - \operatorname{sen} \alpha$$

$$3. \quad (1 + \operatorname{tg}^2 \alpha) (1 - \operatorname{sen}^2 \alpha) = 1$$

$$\left( 1 + \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha} \right) (\cos^2 \alpha) = 1$$

$$\cos^2 \alpha + \operatorname{sen}^2 \alpha = 1$$

$$\frac{\cos^2 \alpha + \operatorname{sen}^2 \alpha}{\cos^2 \alpha} \cdot \cos^2 \alpha = 1$$

$$1 = 1$$

$$4. \quad \cos^2 \alpha \operatorname{tg}^2 \alpha + \operatorname{sen}^2 \alpha \operatorname{ctg}^2 \alpha = 1$$

$$\cos^2 \alpha \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha} + \operatorname{sen}^2 \alpha \frac{\cos^2 \alpha}{\operatorname{sen}^2 \alpha} = 1$$

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1$$

$$1 = 1$$

$$5. \quad (\operatorname{tg} \alpha + \operatorname{ctg} \alpha) \operatorname{sen} \alpha = \frac{1}{\cos \alpha}$$

$$\frac{\operatorname{sen} \alpha}{\cos \alpha} \cdot \operatorname{sen} \alpha + \frac{\cos \alpha}{\operatorname{sen} \alpha} \cdot \operatorname{sen} \alpha = \frac{1}{\cos \alpha}$$

$$\frac{\operatorname{sen}^2 \alpha + \cos^2 \alpha}{\cos \alpha} = \frac{1}{\cos \alpha}$$

$$\frac{\operatorname{sen}^2 \alpha}{\cos \alpha} + \cos \alpha = \frac{1}{\cos \alpha}$$

$$\frac{1}{\cos \alpha} = \frac{1}{\cos \alpha}$$

$$6. \quad \frac{1}{2 - \operatorname{sen}^2 \alpha} = \frac{1 + \operatorname{tg}^2 \alpha}{2 + \operatorname{tg}^2 \alpha}$$

$$\frac{1}{2 - \frac{\operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha}} = \frac{1 + \operatorname{tg}^2 \alpha}{2 + \operatorname{tg}^2 \alpha}$$

$$\frac{1}{\frac{2 + \operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha}} = \frac{1 + \operatorname{tg}^2 \alpha}{2 + \operatorname{tg}^2 \alpha}$$

$$\frac{1}{\frac{2 + 2 \operatorname{tg}^2 \alpha - \operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha}} = \frac{1 + \operatorname{tg}^2 \alpha}{2 + \operatorname{tg}^2 \alpha}$$

$$\frac{1 + \operatorname{tg}^2 \alpha}{2 + \operatorname{tg}^2 \alpha} = \frac{1 + \operatorname{tg}^2 \alpha}{2 + \operatorname{tg}^2 \alpha}$$

$$7. \quad \text{sen}^4 \alpha + \cos^4 \alpha = 1 - 2 \text{ctg}^2 \alpha \text{sen}^4 \alpha$$

$$\text{sen}^4 \alpha + \cos^4 \alpha = 1 - 2 \frac{\cos^2 \alpha}{\text{sen}^2 \alpha} \cdot \text{sen}^4 \alpha$$

$$\text{sen}^4 \alpha + \cos^4 \alpha = 1 - 2 \cos^2 \alpha \text{sen}^2 \alpha$$

$$\text{sen}^4 \alpha + \cos^4 \alpha + 2 \cos^2 \alpha \text{sen}^2 \alpha = 1$$

$$(\text{sen}^2 \alpha + \cos^2 \alpha)^2 = 1$$

$$1 = 1$$

$$8. \quad \text{sen}^2 \alpha + \text{sen}^2 \alpha \cos^2 \alpha + \cos^4 \alpha = 1$$

$$\text{sen}^2 \alpha + \cos^2 \alpha (\text{sen}^2 \alpha + \cos^2 \alpha) = 1$$

$$\text{sen}^2 \alpha + \cos^2 \alpha = 1$$

$$1 = 1$$

Semplifica le seguenti espressioni, usando le formule degli archi associati:

$$9. \quad \frac{\text{sen}(\pi - \alpha) \cos\left(\frac{\pi}{2} + \alpha\right)}{\text{sen}(-\alpha)} + \frac{\text{sen}\left(\frac{\pi}{2} + \alpha\right) \cos\left(\frac{\pi}{2} - \alpha\right)}{\cos(\pi + \alpha)}$$

$$= \frac{\text{sen} \alpha (-\text{sen} \alpha)}{-\text{sen} \alpha} + \frac{\cos \alpha \cdot \text{sen} \alpha}{-\cos \alpha} = \text{sen} \alpha - \text{sen} \alpha = 0$$

$$10. \quad \frac{\text{sen}(-\alpha) + \text{sen}(\pi - \alpha) + \text{sen}(\pi + \alpha)}{\cos(-\alpha) + \cos(\pi - \alpha) + \cos(\pi + \alpha)}$$

$$= \frac{-\text{sen} \alpha + \text{sen} \alpha - \text{sen} \alpha}{\cos \alpha - \cos \alpha - \cos \alpha} = \frac{-\text{sen} \alpha}{-\cos \alpha} = \text{tg} \alpha$$

$$11. \quad \frac{\text{sen}(-\alpha) + \cos(\pi - \alpha) - \text{tg}(\pi + \alpha)}{\text{tg}(\pi - \alpha) - \cos\left(\frac{\pi}{2} - \alpha\right) - \cos(-\alpha)} = \frac{-\text{sen} \alpha - \cos \alpha - \text{tg} \alpha}{-\text{tg} \alpha - \text{sen} \alpha - \cos \alpha} = 1$$

$$12. \quad \frac{\text{sen}\left(\frac{\pi}{2} - \alpha\right) + \cos(-\alpha) + \text{sen}(2\pi - \alpha) + \cos\left(\frac{\pi}{2} - \alpha\right)}{\cos\left(\frac{\pi}{2} + \alpha\right) + \text{sen}(-\alpha)}$$

$$= \frac{\cos \alpha + \cos \alpha - \text{sen} \alpha + \text{sen} \alpha}{-\text{sen} \alpha - \text{sen} \alpha} = \frac{\cos \alpha + \cos \alpha}{-2 \text{sen} \alpha} = \frac{2 \cos \alpha}{-2 \text{sen} \alpha} = -\text{ctg} \alpha$$

Calcola il valore delle seguenti espressioni:

$$\begin{aligned}
 13. \quad & \sqrt{3} \cos \frac{\pi}{3} - 3 \cos \frac{7}{6} \pi + 2 \operatorname{tg} \frac{5}{3} \pi + \sqrt{2} \operatorname{sen} \frac{3}{4} \pi \\
 & = \sqrt{3} \cdot \frac{1}{2} - 3 \left( -\frac{\sqrt{3}}{2} \right) + 2 (-\sqrt{3}) + \sqrt{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} - 2\sqrt{3} + 1 = \mathbf{1}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \operatorname{tg}^2 \frac{7}{6} \pi + \operatorname{sen} \frac{3}{4} \pi \cdot \cos \frac{11}{4} \pi - \frac{1}{3} \cos \frac{4}{3} \pi \\
 & = \left( -\frac{\sqrt{3}}{3} \right)^2 + \frac{\sqrt{2}}{2} \cdot \left( -\frac{\sqrt{2}}{2} \right) - \frac{1}{3} \left( -\frac{1}{2} \right) = \frac{1}{3} - \frac{1}{2} + \frac{1}{6} = \frac{2-3+1}{6} = \mathbf{0}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & 2 \operatorname{sen} \frac{5}{6} \pi + 4 \cos \frac{5}{6} \pi \cdot \operatorname{sen} \frac{2}{3} \pi - 2 \operatorname{tg} \frac{3}{4} \pi + \operatorname{ctg} \frac{5}{6} \pi \\
 & = 2 \cdot \frac{1}{2} + 4 \cdot \left( -\frac{\sqrt{3}}{2} \right) \cdot \frac{\sqrt{3}}{2} - 2 (-1) - \sqrt{3} = 1 - 3 + 2 - \sqrt{3} = \mathbf{-\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \operatorname{tg} \frac{27}{4} \pi - \operatorname{sen} \frac{33}{6} \pi + \cos \frac{45}{4} \pi - \operatorname{tg} \frac{23}{3} \pi + \operatorname{ctg} \frac{29}{6} \pi + \operatorname{sen} \frac{17}{4} \pi \\
 & = -1 - (-1) + \left( -\frac{\sqrt{2}}{2} \right) - (-\sqrt{3}) + (-\sqrt{3}) + \frac{\sqrt{2}}{2} = -1 + 1 - \frac{\sqrt{2}}{2} + \sqrt{3} - \sqrt{3} + \frac{\sqrt{2}}{2} = \mathbf{0}
 \end{aligned}$$