

1. $5^{x+6} = \frac{1}{5^{x-3}}$

$$5^{x+6} = 5^{-x+3}$$

$$x + 6 = -x + 3$$

$$x = -\frac{3}{2}$$

2. $8^{\frac{3+x}{x-1}} = 2^{\frac{15}{2}x}$

$$2^{3 \cdot \frac{3+x}{x-1}} = 2^{\frac{15}{2}x}$$

$$3 \cdot \frac{3+x}{x-1} = \frac{15}{2}x$$

$$\frac{3+x}{x-1} = \frac{5}{2}x$$

$$6 + 2x - 5x^2 + 5x = 0$$

c.a.: $x \neq 1$

$$5x^2 - 7x - 6 = 0$$

$$x_{1,2} = \frac{7 \pm \sqrt{49 + 120}}{10} \begin{cases} 2 \\ -\frac{3}{5} \end{cases}$$

$$x = 2 \quad x = -\frac{3}{5}$$

3. $2^{2x+3} - 17 \cdot 2^x + 2 = 0$

$$2^3 \cdot 2^{2x} - 17 \cdot 2^x + 2 = 0$$

Pongo: $2^x = t$

$$8t^2 - 17t + 2 = 0$$

$$t_{1,2} = \frac{17 \pm \sqrt{289 - 64}}{16} \begin{cases} 2 \\ \frac{1}{8} \end{cases}$$

$$2^x = 2 \quad \Rightarrow \quad x = 1$$

$$2^x = \frac{1}{8} \quad \Rightarrow \quad x = -3$$

4. $25^x - 6 \cdot 5^x + 5 < 0$

$$t^2 - 6t + 5 < 0$$

Pongo $5^x = t$

$$t_{1,2} = \frac{3 \pm \sqrt{9 - 5}}{1} \begin{cases} 5 \\ 1 \end{cases}$$

$$1 < t < 5$$

$$0 < x < 1$$

5. $\frac{5^x - 1}{8 - 2^x} \leq 0$

$$N \geq 0: \quad 5^x - 1 \geq 0 \quad x \geq 0$$

$$D > 0: \quad 8 - 2^x > 0 \quad 2^x < 2^3 \quad x < 3$$

$$x \leq 0 \vee x > 3$$

6. $\ln(2x + 3) = -1$

$$\text{c.a.: } 2x + 3 > 0 \quad x > -\frac{3}{2}$$

$$2x + 3 = e^{-1}$$

$$x = \frac{1}{2e} - \frac{3}{2}$$

$$7. \quad 3 \log_4 (x - 2) - 3 \log_4 (3x + 2) + \log_3 27 - \log_2 8 = 0$$

$$c.a.: \begin{cases} x - 2 > 0 \\ 3x + 2 > 0 \end{cases} \quad \begin{cases} x > 2 \\ x > -\frac{2}{3} \end{cases} \quad x > 2$$

$$3 \log_4 (x - 2) - 3 \log_4 (3x + 2) + 3 - 3 = 0$$

$$3 \log_4 (x - 2) = 3 \log_4 (3x + 2)$$

$$\log_4 (x - 2) = \log_4 (3x + 2)$$

$$-2x = 4$$

$$x = -2 \quad \text{non accettabile} \quad x - 2 = 3x + 2$$

imp.

$$8. \quad \log (4x + 1) < \log (2x - 1) + \log (5 - x)$$

$$\log (4x + 1) < \log (2x - 1)(5 - x)$$

$$\begin{cases} 4x + 1 > 0 \\ 2x - 1 > 0 \\ 5 - x > 0 \\ 4x + 1 < 10x - 2x^2 - 5 + x \end{cases} \quad \begin{cases} x > -\frac{1}{4} \\ x > \frac{1}{2} \\ x < 5 \\ 2x^2 - 7x + 6 < 0 \end{cases}$$

$$x_{1,2} = \frac{7 \pm \sqrt{49 - 48}}{4} \begin{cases} 2 \\ \frac{3}{2} \end{cases}$$

$$\begin{cases} x > -\frac{1}{4} \\ x > \frac{1}{2} \\ x < 5 \\ \frac{3}{2} < x < 2 \end{cases}$$

$$\frac{3}{2} < x < 2$$

$$9. \quad \log_{\frac{1}{3}} (x^2 - 3x) > \log_{\frac{1}{3}} 4$$

$$\begin{cases} x^2 - 3x > 0 \\ x^2 - 3x < 4 \end{cases}$$

$$\begin{cases} x < 0 \vee x > 3 \\ -1 < x < 4 \end{cases}$$

$$x_{1,2} = \frac{3 \pm \sqrt{9 + 16}}{2} \begin{cases} 4 \\ -1 \end{cases}$$

$$-1 < x < 0 \vee 3 < x < 4$$