Calcola il valore delle seguenti espressioni:

1. $\cos 4\pi + 2\ sen\left(-\frac{15}{2}\pi\right) + \frac{1}{3}\cos(-3\pi) + sen\,\frac{9}{2}\pi$

$= 1 + 2\cdot 1 + \frac{1}{3}\cdot(-1) + 1 = 1 + 2 - \frac{1}{3} + 1 = 4 - \frac{1}{3} = \frac{11}{3}$

2. $\dfrac{sen\,\frac{7}{2}\pi - \cos(-7\pi) + 2\ sen\left(-\frac{11}{2}\pi\right)}{2\ sen\left(-\frac{3}{2}\pi\right) + \cos 4\pi - 4\cos\frac{5}{2}\pi} = \dfrac{-1-(-1)+2}{2+1} = \dfrac{2}{3}$

3. $\left[\cos\left(\frac{\pi}{2}\right) - tg\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)\right]\left[sen\left(\frac{\pi}{4}\right) + sen\left(\frac{\pi}{2}\right)\right]$

$= \left(0 - 1 + \frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2} + 1\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

4. $\left[\frac{2}{\sqrt{3}}\,tg\left(\frac{\pi}{3}\right) - \frac{3}{4}\,ctg\left(\frac{\pi}{4}\right) + \cos\pi\right]\left[3 + sen\left(\frac{\pi}{2}\right)\right]$

$= \left(\frac{2}{\sqrt{3}}\sqrt{3} - \frac{3}{4} - 1\right)(3+1) = \frac{1}{4}\cdot 4 = 1$

5. $\left[\sqrt{\dfrac{1-\cos\left(\frac{\pi}{6}\right)}{1+\cos\left(\frac{\pi}{6}\right)}} + \dfrac{tg\left(\frac{\pi}{4}\right) - tg\left(\frac{\pi}{6}\right)}{1 + tg\left(\frac{\pi}{4}\right)tg\left(\frac{\pi}{6}\right)}\right] : \dfrac{1-\cos\left(\frac{\pi}{6}\right)}{sen\left(\frac{\pi}{6}\right)}$

$= \left[\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} + \dfrac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}\right] : \dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} =$

$= \left(\sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}\cdot\dfrac{2-\sqrt{3}}{2-\sqrt{3}}} + \dfrac{3-\sqrt{3}}{3+\sqrt{3}}\cdot\dfrac{3-\sqrt{3}}{3-\sqrt{3}}\right) : (2-\sqrt{3}) =$

$= \left(2 - \sqrt{3} + \dfrac{9 - 6\sqrt{3} + 3}{6}\right)\cdot\dfrac{1}{2-\sqrt{3}} = \dfrac{4-2\sqrt{3}}{2-\sqrt{3}} = 2$

Calcola i valori delle rimanenti funzioni goniometriche, essendo dati:

6. $sen\ \alpha = \dfrac{8}{17} \qquad 0 < \alpha < \dfrac{\pi}{2}$

$$\cos\ \alpha = \sqrt{1 - sen^2\ \alpha} = \sqrt{1 - \frac{64}{289}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$$

$$tg\ \alpha = \frac{sen\ \alpha}{\sqrt{1 - sen^2\ \alpha}} = \frac{8}{17} \cdot \frac{17}{15} = \frac{8}{15} \qquad ctg\ \alpha = \frac{1}{tg\ \alpha} = \frac{15}{8}$$

7. $tg\ \alpha = -\dfrac{7}{24} \qquad \dfrac{\pi}{2} < \alpha < \pi$

$$sen\ \alpha = \frac{tg\ \alpha}{\sqrt{1 + tg^2\ \alpha}} = \frac{7}{24} \cdot \frac{1}{\sqrt{1 + \frac{49}{576}}} = \frac{7}{24} \cdot \frac{24}{25} = \frac{7}{25}$$

$$\cos\ \alpha = -\frac{1}{\sqrt{1 + tg^2\ \alpha}} = -\frac{1}{\sqrt{1 + \frac{49}{576}}} = -\frac{24}{25}$$

$$ctg\ \alpha = \frac{1}{tg\ \alpha} = -\frac{24}{7}$$

Trasforma le seguenti espressioni in altre contenenti solo $sen\ \alpha$:

8. $tg\ \alpha\ ctg\ \alpha - \sec\ \alpha\ \cos\ \alpha + sen\ \alpha \ = 1 - 1 + sen\ \alpha = sen\ \alpha$

9. $(2 + sen\ \alpha\ \cos\ \alpha)\ (2 - sen\ \alpha\ \cos\ \alpha) + \cos^2\ \alpha\ tg^2\ \alpha$

$$= 4 - sen^2\ \alpha\ \cos^2\ \alpha + sen^2\ \alpha = 4 - sen^2\ \alpha\ (1 - sen^2\ \alpha) + sen^2\ \alpha =$$

$$= 4 - sen^2\ \alpha + sen^4\ \alpha + sen^2\ \alpha = 4 + sen^4\ \alpha$$

Trasforma le seguenti espressioni in altre contenenti solo $\cos\ \alpha$:

10. $\sec^2\ \alpha - tg^2\ \alpha + 1$

$$= \frac{1}{\cos^2\ \alpha} - \frac{1 - \cos^2\ \alpha}{\cos^2\ \alpha} + 1 = \frac{1 - 1 + \cos^2\ \alpha + \cos^2\ \alpha}{\cos^2\ \alpha} = \frac{2\ \cos^2\ \alpha}{\cos^2\ \alpha} = 2$$

11. $2\ tg\ \alpha\ sen\ \alpha \left(1 - \dfrac{1}{sen^2\ \alpha}\right)$

$$= 2\ \frac{sen\ \alpha}{\cos\ \alpha}\ sen\ \alpha \left(\frac{sen^2\ \alpha - 1}{sen^2\ \alpha}\right) = \frac{2}{\cos\ \alpha}\ (1 - \cos^2 \alpha - 1) = -2\ \cos\ \alpha$$

Trasforma le seguenti espressioni in altre contenenti solo $tg\ \alpha$:

12. $\dfrac{8}{ctg^2\ \alpha} + tg\ \alpha\ sen^2\ \alpha + ctg\ \alpha\ sen^2\ \alpha = 8\ tg^2\ \alpha + sen^2\ \alpha\ (tg\ \alpha + ctg\ \alpha) =$

$$= 8\ tg^2\ \alpha + sen^2\ \alpha \left(tg\ \alpha + \frac{1}{tg\ \alpha} \right) = 8\ tg^2\ \alpha + \frac{tg^2\ \alpha}{1 + tg^2\ \alpha} \frac{tg^2\ \alpha + 1}{tg\ \alpha} = 8\ tg^2\ \alpha + tg\ \alpha$$