

$$1. \quad 42 \cdot 15^{x-1} + \frac{7 \cdot 15^x}{3} - 6 \cdot 15^{x-1} = 71$$

$$\frac{42 \cdot 15^x}{15} + \frac{7 \cdot 15^x}{3} - \frac{6 \cdot 15^x}{15} = 71$$

$$\frac{71}{15} 15^x = 71$$

$$15^x = 15$$

$$x = 1$$

$$\left( \frac{42}{15} + \frac{7}{3} - \frac{6}{15} \right) 15^x = 71$$

$$2. \quad 9^x - 27 = 26 \cdot 3^x$$

$$\text{Pongo: } 3^x = t \Rightarrow t^2 - 26t - 27 = 0$$

$$t_{1,2} = \frac{13 \pm \sqrt{169 + 27}}{1} \begin{cases} 27 \\ -1 \end{cases}$$

$$t = 27 \Rightarrow 3^x = 27 \Rightarrow x = 3$$

$$9^x - 26 \cdot 3^x - 27 = 0$$

$$t = -1 \Rightarrow 3^x = -1 \Rightarrow \text{imp.}$$

$$3. \quad 5^{x+2} \cdot 7^{x+1} \leq 5$$

$$35^{x+1} \leq 35^0$$

$$x + 1 \leq 0$$

$$x \leq -1$$

$$5 \cdot 35^{x+1} \leq 5$$

$$4. \quad -3^{2x+2} + 3^x > 9^x + 11 \cdot 3^x$$

$$\text{Pongo: } 3^x = t \Rightarrow -9t^2 + t > t^2 + 11t$$

$$-10t^2 - 10t > 0 \Rightarrow t^2 + t < 0$$

$$-1 < t < 0 \Rightarrow -1 < 3^x < 0$$

imp.

$$5. \quad \log_5(x+2) - \log_7(x-4) = \log(3-x)$$

$$\text{c.a.: } \begin{cases} x+2 > 0 \\ x-4 > 0 \\ 3-x > 0 \end{cases} \Rightarrow \begin{cases} x > -2 \\ x > 4 \\ x < 3 \end{cases} \Rightarrow \text{imp.}$$

imp.

$$6. \quad \ln(3x-4) = \ln(x^2-4) - \ln(x-2)$$

$$\text{c.a.: } \begin{cases} 3x-4 > 0 \\ x^2-4 > 0 \\ x-2 > 0 \end{cases} \Rightarrow \begin{cases} x > \frac{4}{3} \\ x < -2 \vee x > 2 \\ x > 2 \end{cases} \Rightarrow x > 2$$

$$\ln(3x-4) = \ln \frac{x^2-4}{x-2}$$

$$3x-4 = x+2$$

$$\ln(3x-4) = \ln(x+2)$$

$$2x = 6$$

$$x = 3$$

7.  $4 \log_4 (2x + 1) = 2$

*c.a.:*  $2x + 1 > 0 \Rightarrow x > -\frac{1}{2}$

$\log_4 (2x + 1) = \frac{1}{2} \quad 2x + 1 = 2$

$x = \frac{1}{2}$

8.  $\log x = \log \sqrt[5]{x^2} + \frac{6}{5}$

*c.a.:*  $x > 0$

$\log x = \frac{2}{5} \log x + \frac{6}{5}$

$\log x - \frac{2}{5} \log x = \frac{6}{5}$

$\frac{3}{5} \log x = \frac{6}{5}$

$\log x = 2$

$x = 100$

9.  $\log_{\frac{3}{2}} (x^2 - 4) - \log_{\frac{3}{2}} (x - 5) > 1$

$\log_{\frac{3}{2}} (x^2 - 4) > \log_{\frac{3}{2}} (x - 5) + 1$

$\log_{\frac{3}{2}} (x^2 - 4) > \log_{\frac{3}{2}} (x - 5) + \log_{\frac{3}{2}} \frac{3}{2}$

$\log_{\frac{3}{2}} (x^2 - 4) > \log_{\frac{3}{2}} \frac{3}{2} (x - 5)$

$x^2 - 4 > \frac{3}{2} (x - 5)$

$$\begin{cases} x^2 - 4 > 0 \\ x - 5 > 0 \\ x^2 - 4 > \frac{3}{2} (x - 5) \end{cases} \Rightarrow \begin{cases} x < -2 \vee x > 2 \\ x > 5 \\ 2x^2 - 8 - 3x + 15 > 0 \end{cases}$$

$$\begin{cases} x < -2 \vee x > 2 \\ x > 5 \\ 2x^2 - 3x + 7 > 0 \end{cases} \Rightarrow x_{1,2} = \frac{3 \pm \sqrt{9 - 56}}{4}$$

$$\begin{cases} x < -2 \vee x > 2 \\ x > 5 \\ \forall x \in R \end{cases}$$

$x > 5$

10.  $\frac{1}{2} \log (x^2 - 5x + 4) < \log x$

$\log (x^2 - 5x + 4) < 2 \log x \Rightarrow \log (x^2 - 5x + 4) < \log x^2$

$$\begin{cases} x^2 - 5x + 4 > 0 \\ x > 0 \\ x^2 - 5x + 4 < x^2 \end{cases} \Rightarrow \begin{cases} x < 1 \vee x > 4 \\ x > 0 \\ x > \frac{4}{5} \end{cases}$$

$\frac{4}{5} < x < 1 \vee x > 4$