B

1. $2^{2(x-2)} - 4^x - (2^x)^2 + 124 = 0$

$\frac{4^x}{16} - 4^x - 4^x + 124 = 0$ $\qquad$ $4^x\left(\frac{1}{16} - 1 - 1\right) = -124$

$\frac{31}{16}\,4^x = 124$ $\qquad$ $4^x = 124 \cdot \frac{16}{31}$

$4^x = 64$ $\qquad$ $x = 3$

2. $4^x + 8 = 9 \cdot 2^x$ $\qquad$ $4^x - 9 \cdot 2^x + 8 = 0$

Pongo: $2^x = t \;\Rightarrow\; t^2 - 9t + 8 = 0$

$t_{1,2} = \frac{9 \pm \sqrt{81-32}}{2} \left\langle \begin{matrix} 8 \\ 1 \end{matrix}\right.$

$t = 8 \;\Rightarrow\; 2^x = 8 \;\Rightarrow\; x = 3$ $\qquad$ $t = 1 \;\Rightarrow\; 2^x = 1 \;\Rightarrow\; x = 0$

3. $3^{x-4} \cdot 7^{x-4} \le 1$

$21^{x-4} \le 21^0$ $\qquad$ $x - 4 \le 0$ $\qquad$ $x \le 4$

4. $-2^{2x} + 2^x > 4^x + 3 \cdot 2^x$

Pongo: $2^x = t \;\Rightarrow\; -t^2 + t > t^2 + 3t$

$-2t^2 - 2t > 0 \;\Rightarrow\; t^2 + t < 0$

$-1 < t < 0 \;\Rightarrow\; -1 < 2^x < 0$ $\qquad$ *imp.*

5. $\log_2(x-3) - \log_2 x = \log_2 x$

$c.a.: \begin{cases} x > 0 \\ x - 3 > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > 3 \end{cases} \Rightarrow x > 3$

$\log_2(x-3) = 2\log_2 x$

$\log_2(x-3) = \log_2 x^2$ $\qquad$ $x - 3 = x^2$

$x^2 - x + 3 = 0$ $\qquad$ $x_{1,2} = \frac{1 \pm \sqrt{1-12}}{1}$ $\qquad$ *imp.*

6. $\log(2x+5) - \log(x+1) = \log 3$

$c.a.: \begin{cases} 2x + 5 > 0 \\ x + 1 > 0 \end{cases} \Rightarrow \begin{cases} x > -\frac{5}{2} \\ x > -1 \end{cases} \Rightarrow x > -1$

$\log(2x+5) = \log(x+1) + \log 3$

$\log(2x+5) = \log 3(x+1)$ $\qquad$ $2x + 5 = 3(x+1)$

$3x + 3 - 2x - 5 = 0$ $\qquad$ $x = 2$

7. $3\log_2(x-2)=3$

$c.a.: x-2>0 \quad \Rightarrow \quad x>2$

$\log_2(x-2)=1 \qquad x-2=2 \qquad \boxed{x=4}$

8. $\ln x-\frac{1}{2}=\ln\sqrt{x}$

$c.a.: x>0$

$\ln x-\frac{1}{2}=\frac{1}{2}\ln x \qquad \ln x-\frac{1}{2}\ln x=\frac{1}{2}$

$\frac{1}{2}\ln x=\frac{1}{2} \qquad \ln x=1 \qquad \boxed{x=e}$

9. $\log_{\frac{1}{5}}(x^2-7)-\log_{\frac{1}{5}}(x-8)>-1$

$\log_{\frac{1}{5}}(x^2-7)>\log_{\frac{1}{5}}(x-8)-1 \qquad \log_{\frac{1}{5}}(x^2-7)>\log_{\frac{1}{5}}(x-8)+\log_{\frac{1}{5}}5$

$\log_{\frac{1}{5}}(x^2-7)>\log_{\frac{1}{5}}5(x-8) \qquad x^2-7<5(x-8)$

$$\begin{cases} x^2-7>0 \\ x-8>0 \\ x^2-7<5(x-8) \end{cases} \Rightarrow \begin{cases} x<-\sqrt{7} \ \vee \ x>\sqrt{7} \\ x>8 \\ x^2-7-5x+40<0 \end{cases}$$

$$\begin{cases} x<-\sqrt{7} \ \vee \ x>\sqrt{7} \\ x>8 \\ x^2-5x+33<0 \end{cases} \Rightarrow x_{1,2}=\frac{5\pm\sqrt{25-132}}{2}$$

$$\begin{cases} x<-\sqrt{7} \ \vee \ x>\sqrt{7} \\ x>8 \\ imp. \end{cases} \qquad \boxed{imp.}$$

10. $\frac{1}{2}\log(-x^2+2x)<\log x$

$\log(-x^2+2x)<2\log x \quad \Rightarrow \quad \log(-x^2+2x)<\log x^2$

$$\begin{cases} -x^2+2x<x^2 \\ -x^2+2x>0 \\ x>0 \end{cases} \Rightarrow \begin{cases} 2x^2-2x>0 \\ x^2-2x<0 \\ x>0 \end{cases} \Rightarrow \begin{cases} x<0 \ \vee \ x>1 \\ 0<x<2 \\ x>0 \end{cases}$$

$$\boxed{1<x<2}$$