

B

$$x = 2$$

$$1. \quad \frac{3^{3x-2}}{9^{x-1}} \cdot \frac{2^{x+1}}{2} = 36$$

$$3^x \cdot 2^x = 36$$

$$\frac{3^{3x-2}}{3^{2x-2}} \cdot 2^x = 36$$

$$6^x = 6^2$$

$$2. \quad 2 \cdot 4^{x+1} + 15 \cdot 2^x - 2 = 0$$

$$\text{Pongo: } 2^x = y$$

$$8y^2 + 15y - 2 = 0$$

$$y_{1,2} = \frac{-15 \pm \sqrt{225 + 64}}{16} = \frac{-15 \pm 17}{16}$$

$$y_1 = \frac{1}{8} \Rightarrow 2^x = 2^{-3} \Rightarrow x = -3$$

$$y_2 = -2 \Rightarrow 2^x = -2 \Rightarrow \text{imp.}$$

$$3. \quad (3^x - 27)(2^x + 7) \geq 0$$

$$3^x - 27 \geq 0 \Rightarrow 3^x \geq 3^3 \Rightarrow x \geq 3$$

$$2^x + 7 \geq 0 \Rightarrow \forall x \in \mathbb{R}$$

$$x \geq 3$$

$$4. \quad \frac{5^{2x+3} - 25}{2^{x-1} - 4} \geq 0$$

$$N \geq 0: 5^{2x+3} - 25 \geq 0 \Rightarrow 5^{2x+3} \geq 25 \Rightarrow 2x+3 \geq 2 \Rightarrow x \geq -\frac{1}{2}$$

$$D > 0: 2^{x-1} - 4 > 0 \Rightarrow 2^{x-1} > 4 \Rightarrow x-1 > 2 \Rightarrow x > 3$$

$$x \leq -\frac{1}{2} \vee x > 3$$

$$5. \quad \left(\frac{1}{9}\right)^x - \frac{28}{3^{2+x}} + \frac{1}{3} > 0$$

$$\frac{1}{9^x} - \frac{28}{3^2 \cdot 3^x} + \frac{1}{3} > 0$$

$$\text{Pongo: } 3^x = y$$

$$\frac{1}{y^2} - \frac{28}{9y} + \frac{1}{3} > 0$$

$$\frac{9 - 28y + 3y^2}{9y^2} > 0$$

$$3y^2 - 28y + 9 > 0$$

$$y_{1,2} = \frac{14 \pm \sqrt{196 - 27}}{3} = \frac{14 \pm 13}{3}$$

$$y < \frac{1}{3} \vee y > 9$$

$$3^x < \frac{1}{3} \vee 3^x > 9$$

$$x < -1 \vee x > 2$$

6. Determina il valore della x nei seguenti casi, utilizzando la definizione di logaritmo:

$$\log_x 2 = \frac{1}{4} \Rightarrow x^{\frac{1}{4}} = 2 \Rightarrow x = 16$$

$$\log_7 x = -1 \Rightarrow 7^{-1} = x \Rightarrow x = \frac{1}{7}$$

$$\log_6 \frac{1}{36} = x \Rightarrow 6^x = \frac{1}{36} \Rightarrow 6^x = 6^{-2} \Rightarrow x = -2$$

$$\log_{(2x-1)} 25 = 2 \Rightarrow (2x-1)^2 = 25 \Rightarrow 2x-1 = 5 \Rightarrow x = 3$$

$$\log_2 (3x+1) = 4 \Rightarrow 2^4 = 3x+1 \Rightarrow 3x = 15 \Rightarrow x = 5$$

$$\log_2 4 = 9x - 7 \Rightarrow 2^{9x-7} = 4 \Rightarrow 9x - 7 = 2 \Rightarrow x = 1$$

$$\log_{(9x-2)} 5 = \frac{1}{2} \Rightarrow (9x-2)^{\frac{1}{2}} = 5 \Rightarrow 9x-2 = 25 \Rightarrow x = 3$$

$$\log_5 (2x-1) = \frac{1}{2} \Rightarrow 5^{\frac{1}{2}} = 2x-1 \Rightarrow 2x = 1 + \sqrt{5} \Rightarrow x = \frac{1 + \sqrt{5}}{2}$$

$$\log_{\frac{1}{2}} 32 = 2x \Rightarrow \left(\frac{1}{2}\right)^{2x} = 32 \Rightarrow \left(\frac{1}{2}\right)^{2x} = \left(\frac{1}{2}\right)^{-5} \Rightarrow x = -\frac{5}{2}$$

$$\log_{3x} 7 = -1 \Rightarrow (3x)^{-1} = 7 \Rightarrow 3x = \frac{1}{7} \Rightarrow x = \frac{1}{21}$$

7. Verifica le seguenti uguaglianze usando le proprietà dei logaritmi:

a. $\log_2 48 - \log_2 3 = 4$ $\log_2 \frac{48}{3} = 4 \Rightarrow \log_2 2^4 = 4 \Rightarrow 4 = 4$

b. $\log_{b^2} (a^2 b^2) = 1 + \frac{\log a}{\log b}$ $\log_{b^2} a^2 + \log_{b^2} b^2 = 1 + \frac{\log a}{\log b}$

$$\frac{\log a^2}{\log b^2} + 1 = 1 + \frac{\log a}{\log b} \Rightarrow \frac{2 \log a}{2 \log b} + 1 = 1 + \frac{\log a}{\log b} \Rightarrow \frac{\log a}{\log b} + 1 = 1 + \frac{\log a}{\log b}$$

c. $\log_a \sqrt{a \sqrt{a \sqrt{a}}} = \frac{7}{8}$ $\frac{1}{2} \log_a a \sqrt{a \sqrt{a}} = \frac{7}{8}$

$$\frac{1}{2} \log_a a + \frac{1}{2} \log_a \sqrt{a \sqrt{a}} = \frac{7}{8} \quad \frac{1}{2} + \frac{1}{4} \log_a a \sqrt{a} = \frac{7}{8}$$

$$\frac{1}{2} + \frac{1}{4} \log_a a + \frac{1}{4} \log_a \sqrt{a} = \frac{7}{8} \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \log_a a = \frac{7}{8} \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$

d. $\log_3 \log_8 \log_2 16 = \log_3 2 - 1$ $\log_3 \log_8 4 = \log_3 2 - 1$

$$\log_3 \frac{\log_2 4}{\log_2 8} = \log_3 2 - 1 \quad \log_3 \frac{2}{3} = \log_3 2 - 1$$

$$\log_3 2 - \log_3 3 = \log_3 2 - 1$$

8. Calcola i seguenti logaritmi, arrotondando il risultato alla quinta cifra decimale:

$$\log_6 812 = 3,73906$$

$$\log_{\frac{3}{4}} 1,3 = -0,91199$$

$$\log_{31} 25 = 0,93736$$