

$$1. \sqrt{27} - 2\sqrt{12} + \sqrt{75} + \sqrt{18} - \sqrt{72} + 3\sqrt{2} = \\ = 3\sqrt{3} - 4\sqrt{3} + 5\sqrt{3} + 3\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = 4\sqrt{3}$$

$$2. \sqrt{8 - 2\sqrt{7}} \cdot \sqrt{8 + 2\sqrt{7}} = \sqrt{64 - 28} = \sqrt{36} = 6$$

$$3. \frac{\sqrt{12}}{\sqrt{12} - \sqrt{2}} - \frac{2}{\sqrt{24} - 2} = \frac{\sqrt{2} \cdot \sqrt{6}}{\sqrt{2}(\sqrt{6} - 1)} - \frac{2}{2(\sqrt{6} - 1)} = \frac{\sqrt{6} - 1}{\sqrt{6} - 1} = 1$$

$$4. \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} + 6 = \frac{\sqrt{6} - 2}{1} - \frac{\sqrt{6} + 2}{1} + 6 = \sqrt{6} - 2 - \sqrt{6} - 2 + 6 = 2$$

$$5. x^2 - x - 2 - \sqrt{2} = 0 \\ \Delta = 1^2 - 4(-2 - \sqrt{2}) = 1 + 8 + 4\sqrt{2} = (2\sqrt{2} + 1)^2 \\ x_{1,2} = \frac{1 \pm (2\sqrt{2} + 1)}{2} \left\langle \begin{array}{l} -\sqrt{2} \\ \sqrt{2} + 1 \end{array} \right.$$

$$6. \frac{x-2}{x-1} + \frac{x-1}{2-x} = \frac{x^2}{x^2 - 3x + 2} \\ \frac{x-2}{x-1} - \frac{x-1}{x-2} = \frac{x^2}{(x-2)(x-1)}$$

c.a.:  $x \neq 1$ ;  $x \neq 2$

$$(x-2)^2 - (x-1)^2 = x^2 \quad \text{eseguo i quadrati e poi, sommando, cambio segno a tutti i termini}$$

$$x^2 + 2x - 3 = 0$$

$$x_{1,2} = -1 \pm 2 \left\langle \begin{array}{l} -3 \\ 1 \end{array} \right. \quad \text{non accettabile per c.a.}$$

$$x = -3$$

$$7. 3x(3x - 2\sqrt{2}) = (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$$

$$3x(3x - 2\sqrt{2}) = 3 \quad \text{divido per 3 entrambi i membri e poi eseguo la moltiplicazione}$$

$$3x^2 - 2\sqrt{2}x - 1 = 0$$

$$\frac{\Delta}{4} = 2 + 3 = 5$$

$$x_{1,2} = \frac{\sqrt{2} \pm \sqrt{5}}{3}$$

$$8. (x - \sqrt{3})^2 + 2(x - \sqrt{3}) - 3 = 0$$

$$\text{Pongo: } x - \sqrt{3} = t$$

$$t^2 + 2t - 3 = 0$$

$$t_{1,2} = -1 \pm 2 \left\langle \begin{array}{l} -3 \\ 1 \end{array} \right.$$

$$\begin{array}{l} x_1 = \sqrt{3} - 3 \\ x_2 = \sqrt{3} + 1 \end{array}$$

$$9. 4x^4 + 3x^2 - 1 = 0 \\ 4t^2 + 3t - 1 = 0$$

$$\text{Pongo: } x^2 = t$$

$$t_{1,2} = \frac{-3 \pm 5}{8} \begin{cases} \frac{1}{4} \\ -1 \text{ non accettabile} \end{cases}$$

$$x_{1,2} = \pm \frac{1}{2}$$

10.  $x(x-3) + \frac{1}{2}x < (2x-5)^2 + \frac{7}{3}x^2$

$$x^2 - 3x + \frac{1}{2}x < 4x^2 - 20x + 25 + \frac{7}{3}x^2$$

$$6x^2 - 18x + 3x < 24x^2 - 120x + 150 + 14x^2$$

$$-32x^2 + 105x - 150 < 0$$

$$32x^2 - 105x + 150 > 0$$

$$\Delta = 105^2 - 4 \cdot 32 \cdot 150 < 0 \Rightarrow \forall x \in R$$

11.  $\begin{cases} x^2 - 4 \leq 0 \\ x(4-x) - 3 \geq 0 \end{cases} \quad A \begin{cases} x^2 - 4 \leq 0 \\ -x^2 + 4x - 3 \geq 0 \end{cases}$

$$A: x_{1,2} = \pm 2 \Rightarrow -2 \leq x \leq 2$$

$$B: x_{1,2} = 2 \pm 1 \Rightarrow 1 \leq x \leq 3$$

$$\Rightarrow 1 \leq x \leq 2$$

12.  $\frac{5x+2}{4-4x} + 1 \geq 0$

$$\frac{x+6}{4(1-x)} \geq 0$$

$$N \geq 0: x \geq -6$$

$$D > 0: x < 1$$

$$\Rightarrow -6 \leq x \leq 1$$

13.  $\frac{1+4x+4x^2}{x^2-x+1} \geq 0$

$$N \geq 0: (1+2x)^2 \geq 0 \Rightarrow \forall x \in R$$

$$D > 0: \Delta < 0 \Rightarrow \forall x \in R$$

$$\Rightarrow \forall x \in R$$

14.  $\begin{cases} x^2 + y^2 = 20 \\ xy = 8 \end{cases} \quad \begin{cases} (x+y)^2 - 2xy = 20 \\ xy = 8 \end{cases}$

$$\begin{cases} (x+y)^2 - 16 = 20 \\ xy = 8 \end{cases}$$

$$\begin{cases} (x+y)^2 = 36 \\ xy = 8 \end{cases}$$

$$\begin{cases} x+y = \pm 6 \\ xy = 8 \end{cases}$$

$$\begin{cases} x+y = 6 \\ xy = 8 \end{cases}$$

$$z^2 - 6z + 8 = 0$$

$$z_{1,2} = 3 \pm 1$$

$$\begin{cases} x=2 \\ y=4 \end{cases}; \begin{cases} x=4 \\ y=2 \end{cases}$$

$$\begin{cases} x+y = -6 \\ xy = 8 \end{cases}$$

$$z^2 + 6z + 8 = 0$$

$$z_{1,2} = -3 \pm 1$$

$$\begin{cases} x=-2 \\ y=-4 \end{cases}; \begin{cases} x=-4 \\ y=-2 \end{cases}$$