VERIFICA DI MATEMATICA

CLASSE 2^D – 26 Febbraio 2007

B

1. $3 \cdot \dfrac{x+1}{1+\sqrt{7}} - \dfrac{x+7}{7+\sqrt{7}} = \dfrac{2x}{\sqrt{7}}$

$3x\sqrt{7} + 3\sqrt{7} - x - 7 = 2x\sqrt{7} + 2x \quad \Rightarrow \quad x\sqrt{7} - 3x = 7 - 3\sqrt{7}$

$x(\sqrt{7} - 3) = \sqrt{7}(\sqrt{7} - 3) \quad \Rightarrow \quad x = \sqrt{7}$

2. $6 + x\sqrt{2} + \dfrac{x}{\sqrt{2}+1} = \dfrac{3x}{\sqrt{2}-1} - 5\sqrt{2}$

$6 + x\sqrt{2} + x\sqrt{2} - x = 3x\sqrt{2} + 3x - 5\sqrt{2} \quad \Rightarrow \quad -x\sqrt{2} - 4x = -5\sqrt{2} - 6$

$x(4 + \sqrt{2}) = 5\sqrt{2} + 6 \quad \Rightarrow \quad x = \dfrac{5\sqrt{2}+6}{4+\sqrt{2}} \cdot \dfrac{4-\sqrt{2}}{4-\sqrt{2}}$

$x = \dfrac{24 - 6\sqrt{2} + 20\sqrt{2} - 10}{14} = \dfrac{14\sqrt{2} + 14}{14} = \sqrt{2} + 1$

3. $\dfrac{3x+\sqrt{3}}{x\sqrt{5}} + \dfrac{\sqrt{5}}{x+\sqrt{3}} = \dfrac{3x+5}{x\sqrt{5}+\sqrt{15}}$ $\qquad$ *c.a.*: $x \neq -\sqrt{3} \ \wedge \ x \neq 0$

$3x^2 + 3x\sqrt{3} + x\sqrt{3} + 3 + 5x = 3x^2 + 5x \quad \Rightarrow \quad 4x\sqrt{3} = -3$

$x = -\dfrac{3}{4\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = -\dfrac{\sqrt{3}}{4}$

4. $\sqrt{3} = \dfrac{\sqrt{15}}{x^2 + x\sqrt{5}} + \dfrac{x\sqrt{3} - \sqrt{5}}{x+\sqrt{5}}$ $\qquad$ *c.a.*: $x \neq 0 \ \wedge \ x \neq -\sqrt{5}$

$\sqrt{3} = \dfrac{\sqrt{15}}{x(x+\sqrt{5})} + \dfrac{x\sqrt{3} - \sqrt{5}}{x+\sqrt{5}} \quad \Rightarrow \quad x^2\sqrt{3} + x\sqrt{15} = \sqrt{15} + x^2\sqrt{3} - x\sqrt{5}$

$x\sqrt{15} + x\sqrt{5} = \sqrt{15} \quad \Rightarrow \quad x\sqrt{5}(\sqrt{3}+1) = \sqrt{15}$

$x = \dfrac{\sqrt{15}}{\sqrt{5}(\sqrt{3}+1)} = \dfrac{\sqrt{3}}{\sqrt{3}+1} \cdot \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{3-\sqrt{3}}{2}$

5. $-\dfrac{(\sqrt{5}+1)(x+1)}{4}-\dfrac{x+2}{\sqrt{5}-2}+\dfrac{2x}{3\sqrt{5}}\geq\dfrac{4}{3}$

$\dfrac{-x\sqrt{5}-\sqrt{5}-x-1}{4}-x\sqrt{5}-2x-2\sqrt{5}-4+\dfrac{2x\sqrt{5}}{15}\geq\dfrac{4}{3}$

$-15x\sqrt{5}-15\sqrt{5}-15x-15-60x\sqrt{5}-120x-120\sqrt{5}-240+8x\sqrt{5}\geq 80$

$-67x\sqrt{5}-135x\geq 335+135\sqrt{5}$

$-x(67\sqrt{5}+135)\geq\sqrt{5}(67\sqrt{5}+135)\qquad\Rightarrow\qquad x\leq-\sqrt{5}$

6. $2+x-\dfrac{6}{3+\sqrt{3}}\leq\dfrac{2-x}{\sqrt{3}-2}+2\cdot\dfrac{3+x}{\sqrt{3}+1}$

$2+x-\dfrac{6}{6}(3-\sqrt{3})\leq-(2-x)(\sqrt{3}+2)+2\cdot\dfrac{3+x}{\sqrt{3}+1}\cdot\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$2+x-3+\sqrt{3}\leq-2\sqrt{3}-4+x\sqrt{3}+2x+3\sqrt{3}-3+x\sqrt{3}-x$

$-2x\sqrt{3}\leq-6\quad\Rightarrow\quad x\sqrt{3}\geq 3\quad\Rightarrow\quad x\geq\dfrac{3}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}\quad\Rightarrow\quad x\geq\sqrt{3}$

7. $\begin{cases}2x-y\sqrt{3}=-\sqrt{15}\\ x(\sqrt{5}+2)-y\sqrt{3}=0\end{cases}$ Applicando il metodo di riduzione e sottraendo le due equazioni:

$2x-x\sqrt{5}+2y=-\sqrt{15}\qquad\Rightarrow\qquad -x\sqrt{5}=-\sqrt{15}\qquad\Rightarrow\qquad x=\sqrt{3}$

Sostituendo nella prima equazione: $2\sqrt{3}-y\sqrt{3}=-\sqrt{15}\qquad\Rightarrow\qquad y\sqrt{3}=\sqrt{3}(\sqrt{5}+2)$

$y=\sqrt{5}+2$

8. $\begin{cases} x\sqrt{7} - y\sqrt{5} = \dfrac{2}{\sqrt{5} - \sqrt{7}} \\ x + y = \dfrac{2}{\sqrt{7} - \sqrt{5}} \end{cases}$

$\begin{cases} x\sqrt{7} - y\sqrt{5} = -(\sqrt{5} + \sqrt{7}) \\ x + y = \sqrt{7} + \sqrt{5} \end{cases}$ Applico il metodo di Cramer:

$$D = \begin{vmatrix} \sqrt{7} & -\sqrt{5} \\ 1 & 1 \end{vmatrix} = \sqrt{7} + \sqrt{5}$$

$$D_x = \begin{vmatrix} -(\sqrt{7} + \sqrt{5}) & -\sqrt{5} \\ \sqrt{7} + \sqrt{5} & 1 \end{vmatrix} = -1\,(\sqrt{7} + \sqrt{5}) + \sqrt{5}\,(\sqrt{7} + \sqrt{5}) = (\sqrt{5} - 1)\,(\sqrt{7} + \sqrt{5})$$

$$D_y = \begin{vmatrix} \sqrt{7} & -(\sqrt{7} + \sqrt{5}) \\ 1 & \sqrt{7} + \sqrt{5} \end{vmatrix} = \sqrt{7}\,(\sqrt{7} + \sqrt{5}) + 1\,(\sqrt{7} + \sqrt{5}) = (\sqrt{7} + 1)\,(\sqrt{7} + \sqrt{5})$$

$$\begin{cases} x = \sqrt{5} - 1 \\ y = \sqrt{7} + 1 \end{cases}$$