ESERCIZI ASSEGNATI PER LE VACANZE NATALIZIE

POTENZA DI UN RADICALE IN R_0^+

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7. $(\sqrt{x+1})^2 = (x+1)^{\frac{1}{2}\cdot 2} = x+1$ $\quad (\sqrt{x+y})^3 = (x+y)^{\frac{1}{2}\cdot 3} = (x+y)\sqrt{x+y}$

$(\sqrt{a^2-b^2})^4 = (a^2-b^2)^{\frac{1}{2}\cdot 4} = (a^2-b^2)^2$ $\quad (\sqrt{3x})^4 = (3x)^{\frac{1}{2}\cdot 4} = (3x)^2 = 9x^2$

8. $(b\sqrt{b-2})^2 = b^2(\sqrt{b-2})^2 = b^2(b-2)$ $\quad (\sqrt{a-b})^4 = (a-b)^{\frac{1}{2}\cdot 4} = (a-b)^2$

$(\sqrt{x+a})^3 = (x+a)^{\frac{1}{2}\cdot 3} = (x+a)\sqrt{x+a}$

9. $(\sqrt[3]{2+ab})^6 = (2+ab)^{\frac{1}{3}\cdot 6} = (2+ab)^2$ $\quad (\sqrt{x-1})^4 = (x-1)^{\frac{1}{2}\cdot 4} = (x-1)^2$

$(\sqrt[3]{a+b})^4 = (a+b)^{\frac{1}{3}\cdot 4} = (a+b)\sqrt[3]{a+b}$

10. $(\sqrt[6]{9ab^2})^4 = (3^{\frac{2}{6}}a^{\frac{1}{6}}b^{\frac{2}{6}})^4 = 3^{\frac{4}{3}}a^{\frac{2}{3}}b^{\frac{4}{3}} = 3b\sqrt[3]{3a^2b}$

$(\sqrt[15]{32ab^3})^5 = (2^{\frac{5}{15}}a^{\frac{1}{15}}b^{\frac{3}{15}})^5 = 2^{\frac{5}{3}}a^{\frac{1}{3}}b = 2b\sqrt[3]{2^2a}$

$(\sqrt[4]{2a^2b^3})^2 = (2^{\frac{1}{4}}a^{\frac{2}{4}}b^{\frac{3}{4}})^2 = 2^{\frac{1}{2}}ab^{\frac{3}{2}} = ab\sqrt{2b}$

11. $(\sqrt[4]{a^3b^5})^3 = (a^{\frac{3}{4}}b^{\frac{5}{4}})^3 = a^{\frac{9}{4}}b^{\frac{15}{4}} = a^2b^3\sqrt[4]{ab^3}$

$(\sqrt[4]{(x-1)^2(x+1)})^2 = \left[(x-1)^{\frac{2}{4}}(x+1)^{\frac{1}{4}}\right]^2 = (x-1)(x+1)^{\frac{1}{2}} = (x-1)\sqrt{x+1}$

$(\sqrt[10]{(x-y)^3})^5 = \left[(x-y)^{\frac{3}{10}}\right]^5 = (x-y)^{\frac{3}{2}} = (x-y)\sqrt{x-y}$

12. $(\sqrt[n]{2\cdot 5^{n-1}})^{2n} = \left(2^{\frac{1}{n}}\cdot 5^{\frac{n-1}{n}}\right)^{2n} = 2^2\cdot 5^{2n-2}$

$(\sqrt[n-1]{2^{n+1}a^2})^2 = \left(2^{\frac{n+1}{n-1}}a^{\frac{2}{n-1}}\right)^2 = 2^{\frac{2n+2}{n-1}}a^{\frac{4}{n-1}} = 2^{2+\frac{4}{n-1}}a^{\frac{4}{n-1}} = 2^2\sqrt[n-1]{2^4a^4}$

$(\sqrt[6n]{xy^2})^{3n^2} = \left(x^{\frac{1}{6n}}y^{\frac{2}{6n}}\right)^{3n^2} = x^{\frac{n}{2}}y^n = y^n\sqrt{x^n}$

13. $(\sqrt[4]{2^na^{3+n}x^3})^{2n} = \left(2^{\frac{n}{4}}a^{\frac{3+n}{4}}x^{\frac{3}{4}}\right)^{2n} = 2^{\frac{n^2}{2}}a^{\frac{3n+n^2}{2}}x^{\frac{3n}{2}} = \sqrt{2^{n^2}a^{3n+n^2}x^{3n}}$

$(\sqrt[6m]{4^ma^{n+1}b})^3 = \left(2^{\frac{2m}{6m}}a^{\frac{n+1}{6m}}b^{\frac{1}{6m}}\right)^3 = 2a^{\frac{n+1}{2m}}b^{\frac{1}{2m}} = 2\sqrt[2m]{a^{n+1}b}$

$(\sqrt[2mn]{5x^2y})^{m^2} = (5x^2y)^{\frac{m^2}{2mn}} = (5x^2y)^{\frac{m}{2n}} = \sqrt[2n]{5^mx^{2m}y^m}$

18. $(2+\sqrt{2})^2 = 4+4\sqrt{2}+2 = 6+4\sqrt{2}$

$(1+\sqrt{2})^2 = 1+2\sqrt{2}+2 = 3+2\sqrt{2}$

$(\sqrt{2}-\sqrt{3})^2 = 2-2\sqrt{6}+3 = 5-2\sqrt{6}$

19. $(2+\sqrt{2})^3 = 8+12\sqrt{2}+12+2\sqrt{2} = 20+14\sqrt{2}$

$(1-\sqrt{3})^3 = 1-3\sqrt{3}+9-3\sqrt{3} = 10-6\sqrt{3}$

$(\sqrt{2}+2\sqrt{3})^3 = 2\sqrt{2}+12\sqrt{3}+36\sqrt{2}+24\sqrt{3} = 38\sqrt{2}+36\sqrt{3}$

20. $(a+\sqrt{a})^2 = a^2+2a\sqrt{a}+a$

$(2\sqrt{a}-a\sqrt{2})^2 = 4a-4a\sqrt{2a}+2a^2$

$(2\sqrt{2}+3)^2 = 8+12\sqrt{2}+9 = 17+12\sqrt{2}$

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21. $(3\sqrt{2}+2\sqrt{3})^2 = 18+12\sqrt{6}+12 = 30+12\sqrt{6}$

$(2-3\sqrt{2})^3 = 8-36\sqrt{2}+108-54\sqrt{2} = 116-90\sqrt{2}$

$(1+\sqrt[3]{2})^3 = 1+3\sqrt[3]{2}+3\sqrt[3]{4}+2 = 3+3\sqrt[3]{2}+3\sqrt[3]{4} = 3(1+\sqrt[3]{2}+\sqrt[3]{4})$

22. $\left(3\sqrt{3}-\dfrac{1}{\sqrt{3}}\right)^2 = 27-2\cdot 3\sqrt{3}\cdot\dfrac{1}{\sqrt{3}}+\dfrac{1}{3} = 27-6+\dfrac{1}{3} = \dfrac{64}{3}$

$\left(\sqrt[6]{\dfrac{4}{5}}+\sqrt{\dfrac{5}{2}}\right)^2 = \left(\sqrt[6]{\dfrac{4}{5}}\right)^2+2\left(\sqrt[6]{\dfrac{4}{5}}\right)\left(\sqrt{\dfrac{5}{2}}\right)+\dfrac{5}{2} = \sqrt[3]{\dfrac{4}{5}}+2\sqrt[6]{\dfrac{25}{2}}+\dfrac{5}{2}$

$\left(\sqrt{\dfrac{a}{b}}-\sqrt{\dfrac{b}{a}}\right)^2 = \dfrac{a}{b}-2\sqrt{\dfrac{a}{b}}\sqrt{\dfrac{b}{a}}+\dfrac{b}{a} = \dfrac{a}{b}-2+\dfrac{b}{a} = \dfrac{a^2-2ab+b^2}{ab} = \dfrac{(a-b)^2}{ab}$

23. $\left(2\sqrt{2}-\dfrac{1}{\sqrt{2}}\right)^2 = 8-2\cdot 2\sqrt{2}\cdot\dfrac{1}{\sqrt{2}}+\dfrac{1}{2} = 8-4+\dfrac{1}{2} = \dfrac{9}{2}$

$\left(1-\sqrt[6]{5}\right)^2 = 1-2\sqrt[6]{5}+\sqrt[3]{5}$

$(\sqrt[3]{2}+2\sqrt[3]{4})^2 = \sqrt[3]{4}+4\sqrt[3]{2}\cdot\sqrt[3]{4}+4\sqrt[3]{2^4} = \sqrt[3]{4}+8+8\sqrt[3]{2}$

ESTRAZIONE DI RADICE DA UN RADICALE IN R_0^+

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1. $\sqrt[3]{\sqrt{5}} = \left(5^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{6}} = \sqrt[6]{5}$ $\sqrt{\sqrt{3a}} = \left((3a)^{\frac{1}{2}}\right)^{\frac{1}{2}} = (3a)^{\frac{1}{4}} = \sqrt[4]{3a}$

$\sqrt{\sqrt[3]{3a^2b}} = \left((3a^2b)^{\frac{1}{3}}\right)^{\frac{1}{2}} = \sqrt[6]{3a^2b}$ $\sqrt[4]{\sqrt[3]{12}} = \left((12)^{\frac{1}{3}}\right)^{\frac{1}{4}} = \sqrt[12]{12}$

2. $\sqrt[3]{\sqrt[3]{81}} = \left((81)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \sqrt[9]{81}$ $\sqrt[3]{\sqrt[5]{8}} = \left((2^3)^{\frac{1}{5}}\right)^{\frac{1}{3}} = \sqrt[5]{2}$

$\sqrt[3]{\sqrt[3]{2a^2}} = \left((2a^2)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \sqrt[9]{2a^2}$ $\sqrt[3]{\sqrt[5]{16a^2}} = \left((16a^2)^{\frac{1}{5}}\right)^{\frac{1}{3}} = \sqrt[15]{16a^2}$

6. $\sqrt[3]{4\sqrt{2}} = \sqrt[3]{2^2 \cdot 2^{\frac{1}{2}}} = \sqrt[3]{2^{\frac{3}{2}}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{3}} = 2^{\frac{1}{2}} = \sqrt{2}$

$\sqrt{3\sqrt[3]{9}} = \sqrt{3 \cdot 3^{\frac{2}{3}}} = \sqrt{3^{\frac{5}{3}}} = \left(3^{\frac{5}{3}}\right)^{\frac{1}{2}} = \sqrt[6]{3^5}$

$\sqrt{2\sqrt[3]{2a^2}} = \sqrt{2 \cdot 2^{\frac{1}{3}}\, a^{\frac{2}{3}}} = \left(2^{\frac{4}{3}}\, a^{\frac{2}{3}}\right)^{\frac{1}{2}} = 2^{\frac{2}{3}}\, a^{\frac{1}{3}} = \sqrt[3]{4a}$

$\sqrt[3]{a^2\sqrt[5]{\frac{1}{a^4}}} = \sqrt[3]{a^2 \cdot \frac{1}{a^{\frac{4}{5}}}} = \sqrt[3]{a^{\frac{6}{5}}} = \left(a^{\frac{6}{5}}\right)^{\frac{1}{3}} = a^{\frac{2}{5}} = \sqrt[5]{a^2}$

$\sqrt{x\sqrt{x\sqrt{x}}} = \sqrt{x\sqrt{x \cdot x^{\frac{1}{2}}}} = \sqrt{x\sqrt{x^{\frac{3}{2}}}} = \sqrt{x \cdot x^{\frac{3}{4}}} = \sqrt{x^{\frac{7}{4}}} = x^{\frac{7}{8}} = \sqrt[8]{x^7}$