

ESERCIZI ASSEGNATI PER LE VACANZE NATALIZIE

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$$44. \sqrt{2x^4} + \sqrt{8x^2y^2} + \sqrt{2y^4} = x^2\sqrt{2} + 2xy\sqrt{2} + y^2\sqrt{2} = (x^2 + 2xy + y^2)\sqrt{2} = (x+y)^2\sqrt{2}$$

$$49. \sqrt{3a^2} \cdot \sqrt[4]{3a} + \sqrt[4]{27ab^2} \cdot \sqrt{b} = 3^{\frac{1}{2}}a \cdot 3^{\frac{1}{4}}a^{\frac{1}{4}} + 3^{\frac{3}{4}}a^{\frac{1}{4}}b^{\frac{2}{4}}b^{\frac{1}{2}} = 3^{\frac{3}{4}}a^{\frac{5}{4}}(a+b) = (a+b)\sqrt[4]{27a}$$

$$\sqrt{a^5} + \sqrt{4ab^2} - \sqrt{4a^3} - \sqrt{a^3b^2} = a^2\sqrt{a} + 2b\sqrt{a} - 2a\sqrt{a} - ab\sqrt{a} = (a-2)(a-b)\sqrt{a}$$

$$51. \sqrt[3]{x^4 - x^3y} + \sqrt[3]{xy^3 - y^4} - \sqrt[3]{x-y} = \sqrt[3]{x-y}(x+y-1)$$

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$$1. \sqrt{2}x = \sqrt{8} \quad x = \frac{\sqrt{8}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}} = 2$$

$$\sqrt{2}x = \sqrt{6} \quad x = \frac{\sqrt{6}}{\sqrt{2}} = \frac{2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{2^{\frac{1}{2}}} = \sqrt{3}$$

$$x + \sqrt{3} = 2(x + \sqrt{3}) \quad x - 2x = -\sqrt{3} + 2\sqrt{3} \quad x = -\sqrt{3}$$

$$3. \sqrt{8}x + \sqrt{18}x = 5 \quad 2\sqrt{2}x + 3\sqrt{2}x = 5 \quad x = \frac{1}{\sqrt{2}}$$

$$\sqrt{3}x = \sqrt{6} + \sqrt{27} \quad \sqrt{3}x = \sqrt{6} + 3\sqrt{3} \quad x = \sqrt{2} + 3$$

$$2x + 2\sqrt{3} = \sqrt{3}x + 4 \quad x(2 - \sqrt{3}) = 4 - 2\sqrt{3} \quad x = 2$$

$$5. \sqrt{3}(x - \sqrt{6} + 1) = \sqrt{3}(1 - x) + \sqrt{18} \quad \sqrt{3}x - 3\sqrt{2} + \sqrt{3} = \sqrt{3} - \sqrt{3}x + 3\sqrt{2}$$

$$2\sqrt{3}x = 6\sqrt{2} \quad x = \frac{6\sqrt{2}}{2\sqrt{3}} = \frac{3\sqrt{2}}{\sqrt{3}} = \frac{3 \cdot 2^{\frac{1}{2}}}{3^{\frac{1}{2}}} = 3^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$$

$$6. x(\sqrt{5} - 5) + \sqrt{2}(x + 1) - \sqrt{2} = \sqrt{125} + x(\sqrt{2} - 5)$$

$$x\sqrt{5} - 5x + x\sqrt{2} + \sqrt{2} - \sqrt{2} = 5\sqrt{5} + x\sqrt{2} - 5x \quad x = 5$$

$$x\sqrt{3} = x + 2\sqrt{3} \quad x(\sqrt{3} - 1) = 2\sqrt{3} \quad x = \frac{2\sqrt{3}}{\sqrt{3} - 1}$$

$$x = \frac{2\sqrt{3}}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2(3 + \sqrt{3})}{3 - 1} = 3 + \sqrt{3}$$

$$7. \sqrt{2}x = 2 + \sqrt{2} \quad x = \frac{2 + \sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(\sqrt{2} + 1)}{\sqrt{2}} = \sqrt{2} + 1$$

$$2\sqrt{2}x + 3 + \sqrt{2} = \sqrt{3}x + \sqrt{2}(2\sqrt{3} + 1) \quad 2\sqrt{2}x + 3 + \sqrt{2} = \sqrt{3}x + 2\sqrt{6} + \sqrt{2}$$

$$x(2\sqrt{2} - \sqrt{3}) = 2\sqrt{6} - 3 \quad x = \frac{\sqrt{3}(2\sqrt{2} - \sqrt{3})}{(2\sqrt{2} - \sqrt{3})} = \sqrt{3}$$

$$8. \quad (2 - \sqrt{3})x - \sqrt{6} = 2x - \sqrt{3} (2\sqrt{2} + 1) \qquad 2x - \sqrt{3}x - \sqrt{6} = 2x - 2\sqrt{6} - \sqrt{3}$$

$$-\sqrt{3}x = -\sqrt{3}(\sqrt{2} + 1) \qquad x = \sqrt{2} + 1$$

$$x(3 - \sqrt{3}) = 6 \left(\frac{x}{3 - \sqrt{3}} - 2 \right) \qquad \frac{x(9 + 3 - 6\sqrt{3})}{3 - \sqrt{3}} = \frac{6x - 36 + 12\sqrt{3}}{3 - \sqrt{3}}$$

$$6x(1 - \sqrt{3}) = 12(-3 + \sqrt{3}) \qquad x = \frac{12\sqrt{3}(-\sqrt{3} + 1)}{6(1 - \sqrt{3})} = 2\sqrt{3}$$

$$9. \quad \frac{x - 2\sqrt{2} - 4}{6\sqrt{2}} + \frac{x + 2\sqrt{2}}{2\sqrt{2}} = \frac{x + 1}{3} \qquad \frac{x - 2\sqrt{2} - 4 + 3x + 6\sqrt{2}}{6\sqrt{2}} = \frac{2\sqrt{2}x + 2\sqrt{2}}{6\sqrt{2}}$$

$$4x - 2\sqrt{2}x = 4 - 2\sqrt{2} \qquad x = 1$$

$$\frac{x - 2}{\sqrt{3} - 1} + \frac{2x}{\sqrt{3} + 1} = 3 \qquad \frac{\sqrt{3}x + x - 2\sqrt{3} - 2 + 2x\sqrt{3} - 2x - 6}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = 0 \qquad 3x\sqrt{3} - x = 8 + 2\sqrt{3}$$

$$x = \frac{8 + 2\sqrt{3}}{3\sqrt{3} - 1} \cdot \frac{3\sqrt{3} + 1}{3\sqrt{3} + 1} = \frac{24\sqrt{3} + 8 + 18 + 2\sqrt{3}}{27 - 1} = \frac{26(\sqrt{3} + 1)}{26} = \sqrt{3} + 1$$

$$11. \quad \frac{3(x - 3\sqrt{2})}{4\sqrt{3}} - \frac{x - 2\sqrt{3}}{3\sqrt{2}} = \frac{1}{2} \qquad \frac{9\sqrt{2}x - 54 - 4\sqrt{3}x + 24 - 6\sqrt{6}}{12\sqrt{6}} = 0$$

$$x(9\sqrt{2} - 4\sqrt{3}) = 30 + 6\sqrt{6}$$

$$x = \frac{30 + 6\sqrt{6}}{9\sqrt{2} - 4\sqrt{3}} \cdot \frac{9\sqrt{2} + 4\sqrt{3}}{9\sqrt{2} + 4\sqrt{3}} = \frac{6(45\sqrt{2} + 20\sqrt{3} + 18\sqrt{3} + 12\sqrt{2})}{162 - 48}$$

$$= \frac{57\sqrt{2} + 38\sqrt{3}}{19} = \frac{19(3\sqrt{2} + 2\sqrt{3})}{19} = 3\sqrt{2} + 2\sqrt{3}$$

$$\frac{6}{7} + \frac{x}{3 - \sqrt{2}} = 2 - \frac{x}{7}(\sqrt{2} - 3) \qquad \frac{18 - 6\sqrt{2} + 7x - 42 + 14\sqrt{2} + 6\sqrt{2}x - 11x}{7(3 - \sqrt{2})} = 0$$

$$6\sqrt{2}x - 4x = -8\sqrt{2} + 24 \qquad x = \frac{8(3 - \sqrt{2})}{2\sqrt{2}(3 - \sqrt{2})} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$$

$$12. \quad 1 - \frac{x - 2 + \sqrt{2}}{2 - \sqrt{2}} = \frac{2 + \sqrt{2} - x}{2 + \sqrt{2}}$$

$$\frac{2 - 2x - x\sqrt{2} + 4 + 2\sqrt{2} - 2\sqrt{2} - 2}{(2 - \sqrt{2})(2 + \sqrt{2})} = \frac{4 - 2\sqrt{2} + 2\sqrt{2} - 2 - 2x + x\sqrt{2}}{(2 - \sqrt{2})(2 + \sqrt{2})}$$

$$-2x\sqrt{2} = -2 \qquad x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{x} + \frac{1}{x - \sqrt{2}} = \frac{\sqrt{2}}{x} \qquad \frac{x - \sqrt{2} + x - x\sqrt{2} + 2}{x(x - \sqrt{2})} = 0 \qquad x \neq 0; \sqrt{2}$$

$$2x - x\sqrt{2} = \sqrt{2} - 2 \qquad x = -1$$

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$$6. \frac{3\sqrt{8} + \sqrt{125} - 2\sqrt{18} + 4\sqrt{50} - \sqrt{45}}{\sqrt{5} + 10\sqrt{2}} = \frac{6\sqrt{2} + 5\sqrt{5} - 6\sqrt{2} + 20\sqrt{2} - 3\sqrt{5}}{\sqrt{5} + 10\sqrt{2}} =$$

$$= \frac{2\sqrt{5} + 20\sqrt{2}}{\sqrt{5} + 10\sqrt{2}} = \frac{2(\sqrt{5} + 10\sqrt{2})}{\sqrt{5} + 10\sqrt{2}} = 2$$

$$7. \left[\frac{\sqrt{3}}{\sqrt{12}} - 2\sqrt{3} + \frac{2}{3}\sqrt{\frac{3}{8}} : \frac{\sqrt{3}}{\sqrt{2}} - \sqrt{\left(\frac{1}{\sqrt[4]{3}}\right)^8} + \frac{\sqrt{150}}{\sqrt{2}} \right]^2 - \frac{109}{4} =$$

$$= \left[\frac{\sqrt{3}}{2\sqrt{3}} - 2\sqrt{3} + \frac{2}{3} \cdot \frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{3}} - \sqrt{\frac{1}{9}} + \frac{5\sqrt{6}}{\sqrt{2}} \right]^2 - \frac{109}{4} =$$

$$= \left[\frac{1}{2} - 2\sqrt{3} + \frac{1}{3} - \frac{1}{3} + 5\sqrt{3} \right]^2 - \frac{109}{4} = 27 + \frac{1}{4} + 3\sqrt{3} - \frac{109}{4} = 3\sqrt{3}$$

$$12. [(2\sqrt{5} - 3\sqrt{10})^2 - (2\sqrt{2} + 1)^2 + (\sqrt{3} - \sqrt{6})^2] (11 + 7\sqrt{2}) =$$

$$= [20 + 90 - 60\sqrt{2} - 8 - 1 - 4\sqrt{2} + 3 + 6 - 6\sqrt{2}] (11 + 7\sqrt{2}) =$$

$$= [110 - 70\sqrt{2}] (11 + 7\sqrt{2}) = 10(11 - 7\sqrt{2})(11 + 7\sqrt{2}) = 10(121 - 98) = 230$$

$$15. \left(\sqrt{2\sqrt{5}} \right)^2 + (2 + \sqrt{5})^2 + (3 - \sqrt{5})^2 + (\sqrt{7} - 7\sqrt{2})(\sqrt{7} + 7\sqrt{2}) =$$

$$= 2\sqrt{5} + 4 + 5 + 4\sqrt{5} + 9 + 5 - 6\sqrt{5} + 7 - 98 = -68$$

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$$23. \frac{1}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{5} - \sqrt{3}} - \frac{3}{2}\sqrt{\frac{20}{9}} = \frac{\sqrt{5} - \sqrt{3} + \sqrt{5} + \sqrt{3}}{2} - \frac{3}{2} \cdot \frac{2\sqrt{5}}{3} = \sqrt{5} - \sqrt{5} = 0$$

$$29. (\sqrt{2} + \sqrt[3]{5})^2 - (2\sqrt{5} - \sqrt[3]{2})^2 + (\sqrt[3]{5} + \sqrt[3]{2})^2 - 2(\sqrt[6]{200} + 2\sqrt[6]{500}) =$$

$$= 2 + \sqrt[3]{25} + 2\sqrt[6]{200} - 20 - \sqrt[3]{4} + 4\sqrt[6]{500} + \sqrt[3]{25} + \sqrt[3]{4} + 2\sqrt[3]{10} - 2\sqrt[6]{200} - 4\sqrt[6]{500} =$$

$$= 2\sqrt[3]{25} - 18 + 2\sqrt[3]{10}$$

$$3. (\sqrt{a+1} - \sqrt{a-2})(\sqrt{a+1} + \sqrt{a-2}) = a+1 - a+2 = 3$$

$$6. \sqrt[4]{\frac{a^3 + 3a^2 + 3a + 1}{a^2 - 2a + 1}} : \sqrt[3]{\frac{a+1}{(a-1)^2}} : \sqrt[6]{\frac{a^2+a}{a-1}} =$$

$$= \frac{(a+1)^{\frac{3}{4}}}{(a-1)^{\frac{2}{4}}} \cdot \frac{(a-1)^{\frac{2}{3}}}{(a+1)^{\frac{1}{3}}} \cdot \frac{(a-1)^{\frac{1}{6}}}{a^{\frac{1}{6}}(a+1)^{\frac{1}{6}}} = \frac{(a+1)^{\frac{1}{4}}(a-1)^{\frac{1}{3}}}{a^{\frac{1}{6}}} = \sqrt[12]{\frac{(a+1)^3(a-1)^4}{a^2}}$$

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$$13. \frac{\sqrt{b}}{\sqrt{a} + \sqrt{b}} - \frac{\sqrt{b}}{\sqrt{a} - \sqrt{b}} + \frac{2a}{a-b} = \frac{\sqrt{ab} - b - \sqrt{ab} - b + 2a}{a-b} = \frac{2(a-b)}{a-b} = 2$$

$$\left(\sqrt[3]{a} \sqrt[5]{a^2} \sqrt{a} \right)^2 = \left(\sqrt[3]{a} \sqrt[5]{a^2} a^{\frac{1}{2}} \right)^2 = \left(\sqrt[3]{a} \sqrt[5]{a^{\frac{5}{2}}} \right)^2 = \left(\sqrt[3]{a} a^{\frac{1}{2}} \right)^2 = \left(\sqrt[3]{a^{\frac{3}{2}}} \right)^2 = \left(a^{\frac{1}{2}} \right)^2 = a$$

$$15. \frac{3\sqrt{a} + 2\sqrt{x}}{3\sqrt{a} - 2\sqrt{x}} - \frac{3\sqrt{a} - 2\sqrt{x}}{3\sqrt{a} + 2\sqrt{x}} - \frac{24\sqrt{ax}}{9a - 4x} =$$

$$= \frac{9a + 4x + 12\sqrt{ax} - 9a - 4x + 12\sqrt{ax} - 24\sqrt{ax}}{9a - 4x} = 0$$

$$\frac{\sqrt[4]{1-x} + \frac{1}{\sqrt[4]{1+x}}}{1 + \frac{1}{\sqrt[4]{1-x^2}}} - \frac{\sqrt{1-x}}{\sqrt[4]{1-x}} = \frac{\sqrt[4]{1-x^2} + 1}{\sqrt[4]{1-x^2}} - \frac{\sqrt{1-x}}{\sqrt[4]{1-x}} = \frac{\sqrt[4]{1-x^2}}{\sqrt[4]{1-x^2}} - \frac{(1-x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{4}}} =$$

$$= \frac{(1-x)^{\frac{1}{4}} (1+x)^{\frac{1}{4}}}{(1+x)^{\frac{1}{4}}} - (1-x)^{\frac{1}{4}} = \sqrt[4]{1-x} - \sqrt[4]{1-x} = 0$$

$$21. \sqrt{\frac{1-a^3}{3-3a}} : \left(\sqrt{\frac{a+a^2+a^3}{1+a}} : \sqrt{3a^3} \right) = \frac{(1-a)^{\frac{1}{2}} (1+a+a^2)^{\frac{1}{2}}}{3^{\frac{1}{2}} (1-a)^{\frac{1}{2}}} \cdot \frac{(1+a)^{\frac{1}{2}}}{a^{\frac{1}{2}} (1+a+a^2)^{\frac{1}{2}}} \cdot 3^{\frac{1}{2}} a^{\frac{3}{2}} =$$

$$= (1+a)^{\frac{1}{2}} \cdot a = a \sqrt{1+a}$$

$$22. \sqrt{\frac{a^2+ab}{xy}} \cdot \sqrt[4]{\frac{ab+b^2}{x}} : \sqrt{\frac{a+b}{xy}} : \sqrt[4]{\frac{a^3b^2+a^2b^3}{x^2}} =$$

$$= \frac{a^{\frac{1}{2}} (a+b)^{\frac{1}{2}}}{x^{\frac{1}{2}} y^{\frac{1}{2}}} \cdot \frac{b^{\frac{1}{4}} (a+b)^{\frac{1}{4}}}{x^{\frac{1}{4}}} \cdot \frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{(a+b)^{\frac{1}{2}}} \cdot \frac{x^{\frac{2}{4}}}{a^{\frac{2}{4}} b^{\frac{2}{4}} (a+b)^{\frac{1}{4}}} = \frac{x^{\frac{1}{4}}}{b^{\frac{1}{4}}} = \sqrt[4]{\frac{x}{b}}$$

$$23. \sqrt[3]{x} \sqrt{x} \cdot \sqrt{x \sqrt{x} \sqrt[3]{x}} : \sqrt[3]{x} = \sqrt[3]{x^{\frac{3}{2}}} \cdot \sqrt{x \sqrt{x} \sqrt[3]{x}} : x^{\frac{1}{3}} = x^{\frac{1}{2}} \cdot \sqrt{x^{\frac{5}{3}}} : x^{\frac{1}{3}} = x^{\frac{1}{2} + \frac{5}{6} - \frac{1}{3}} = x$$