

ESERCIZI ASSEGNATI PER LE VACANZE NATALIZIE

Pag. 138

$$11. \sqrt{\sqrt[3]{2}} \cdot \sqrt[4]{2} = \left(2^{\frac{1}{3}}\right)^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} = 2^{\frac{1}{6}} \cdot 2^{\frac{1}{4}} = 2^{\frac{1}{6} + \frac{1}{4}} = 2^{\frac{5}{12}} = \sqrt[12]{2^5}$$

$$4\sqrt[5]{\sqrt[6]{6}} \cdot \sqrt{\sqrt[5]{4}} : \sqrt{\sqrt[3]{3}} = \left((2 \cdot 3)^{\frac{1}{5}}\right)^{\frac{1}{4}} \cdot \left(2^{\frac{2}{5}}\right)^{\frac{1}{2}} : \left(3^{\frac{1}{2}}\right)^{\frac{1}{10}} = 2^{\frac{1}{20}} \cdot 3^{\frac{1}{20}} \cdot 2^{\frac{1}{5}} : 3^{\frac{1}{20}} = 2^{\frac{5}{20}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

$$\frac{4\sqrt[3]{\sqrt[3]{5}}}{\sqrt[3]{\sqrt[3]{5}}} = \left(5^{\frac{1}{3}}\right)^{\frac{1}{4}} : \left(5^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{12}} : 5^{\frac{1}{6}} = 5^{-\frac{1}{12}} = \frac{1}{\sqrt[12]{5}}$$

$$\frac{\sqrt{\sqrt[5]{8}}}{\sqrt[5]{\sqrt[3]{2}}} = \left(2^{\frac{3}{5}}\right)^{\frac{1}{2}} : \left(2^{\frac{1}{3}}\right)^{\frac{1}{5}} = 2^{\frac{3}{10}} : 2^{\frac{1}{15}} = 2^{\frac{7}{30}} = \sqrt[30]{2^7}$$

$$12. \sqrt[6]{5} \cdot \sqrt[4]{\sqrt{10}} \cdot \sqrt[3]{\sqrt{2}} = 5^{\frac{1}{6}} \cdot \left((2 \cdot 5)^{\frac{1}{2}}\right)^{\frac{1}{4}} \cdot \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{6}} \cdot 2^{\frac{1}{8}} \cdot 5^{\frac{1}{8}} \cdot 2^{\frac{1}{6}} = 2^{\frac{7}{24}} \cdot 5^{\frac{7}{24}} = 10^{\frac{7}{24}} = \sqrt[24]{10^7}$$

$$\frac{\sqrt[5]{\sqrt[3]{3}} \cdot \sqrt[3]{\sqrt{2}}}{\sqrt[6]{\sqrt[5]{6}}} = \left(3^{\frac{1}{3}}\right)^{\frac{1}{5}} \cdot \left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} : \left((2 \cdot 3)^{\frac{1}{5}}\right)^{\frac{1}{6}} = 3^{\frac{1}{15} - \frac{1}{30}} \cdot 2^{\frac{1}{6} - \frac{1}{30}} = 3^{\frac{1}{30}} \cdot 2^{\frac{4}{30}} = \sqrt[30]{3 \cdot 2^4} = \sqrt[30]{48}$$

$$\sqrt{x^2} \sqrt[7]{x} \cdot \sqrt{x^3} \sqrt{x} : (4\sqrt{x})^2 = \left(x^2 \cdot x^{\frac{1}{7}}\right)^{\frac{1}{5}} \cdot \left(x^3 \cdot x^{\frac{1}{2}}\right)^{\frac{1}{7}} : \left(x^4\right)^2 = x^{\frac{3}{7}} \cdot x^{\frac{1}{2}} : x^2 = x^{\frac{3}{7}} = \sqrt[7]{x^3}$$

$$13. \sqrt{\frac{1}{2}\sqrt{2}} \cdot \sqrt{\frac{1}{2}\sqrt[3]{4}} \cdot \sqrt[3]{\sqrt{2 + \frac{1}{2}}} = \left(\frac{1}{2} \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left(\frac{1}{2} \cdot 2^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot \left(\left(\frac{5}{2}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \frac{1}{2^{\frac{1}{4}}} \cdot \frac{1}{2^{\frac{1}{6}}} \cdot \frac{5^{\frac{1}{6}}}{2^{\frac{1}{6}}} = \frac{5^{\frac{1}{6}}}{2^{\frac{7}{12}}} = \sqrt[12]{\frac{5^2}{2^7}}$$

$$\sqrt{a\sqrt{a}} \cdot \sqrt[3]{a\sqrt{a}} = \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}} \cdot \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{3}} = a^{\frac{3}{4}} \cdot a^{\frac{1}{2}} = a^{\frac{5}{4}} = a\sqrt[4]{a}$$

$$18. \sqrt[4]{3x^2y} \sqrt{2xy} \sqrt[3]{2xy^3} : \sqrt[12]{108} = \sqrt[4]{3x^2y} \sqrt{2xy} \cdot 2^{\frac{1}{3}} x^{\frac{1}{3}} y^{\frac{3}{3}} : \sqrt[12]{2^2 \cdot 3^3} = \sqrt[4]{3x^2y} \cdot 2^{\frac{2}{3}} x^{\frac{2}{3}} y^{\frac{1}{2}} : \left(2^{\frac{1}{6}} \cdot 3^{\frac{1}{4}}\right) = x^{\frac{4}{6}} y^{\frac{3}{6}} = \sqrt[6]{x^4 y^3}$$

$$19. \sqrt[3]{a^2b} \sqrt{5a^2} : \left(\sqrt[3]{a\sqrt{a+b}} \cdot \sqrt{(a+b)\sqrt[3]{a\sqrt{5}}}\right)^2 = \sqrt[3]{a^2b} \cdot 5^{\frac{1}{2}} a^{\frac{2}{2}} : \left(\sqrt[3]{a(a+b)^{\frac{1}{2}}} \cdot \sqrt{(a+b)a^{\frac{1}{3}}5^{\frac{1}{6}}}\right)^2 = 5^{\frac{1}{6}} a^{\frac{3}{3}} b^{\frac{1}{3}} : \left(a^{\frac{1}{3}}(a+b)^{\frac{1}{6}}(a+b)^{\frac{1}{2}}a^{\frac{1}{6}}5^{\frac{1}{12}}\right)^2 =$$

$$= 5^{\frac{1}{6}} a^{\frac{1}{3}} : \left((a+b)^{\frac{2}{3}} a^{\frac{1}{2}} 5^{\frac{1}{12}} \right)^2 = \frac{5^{\frac{1}{6}} a^{\frac{1}{3}}}{(a+b)^{\frac{4}{3}} a^{\frac{1}{6}} 5^{\frac{1}{6}}} = \frac{1}{a+b} \sqrt[3]{\frac{b}{a+b}}$$

$$20. \sqrt{a} \sqrt[3]{a} \sqrt[3]{a^2} \cdot \sqrt[3]{a} \sqrt[3]{\frac{1}{a}} : \sqrt{\frac{1}{a}} = \sqrt{a} \sqrt[3]{a \cdot a^{\frac{2}{3}}} \cdot \sqrt[3]{a \cdot \frac{1}{a^{\frac{1}{3}}}} = \sqrt{a \cdot a^{\frac{5}{9}}} \cdot \sqrt[3]{a^{\frac{2}{3}}} =$$

$$= \sqrt{a^{\frac{14}{9}}} \cdot a^{\frac{7}{18}} = a^{\frac{7}{9}} \cdot a^{\frac{7}{18}} = a^{\frac{21}{18}} = a^{\frac{7}{6}} = a^{1 + \frac{1}{6}} = a \sqrt[6]{a}$$

$$\sqrt[5]{x} \sqrt[7]{x^3} \cdot \sqrt{x} \sqrt[7]{\frac{1}{x^2}} : \sqrt[7]{x^4} \sqrt{x} = \sqrt[5]{x \cdot x^{\frac{3}{7}}} \cdot \sqrt{x \cdot \frac{1}{x^{\frac{2}{7}}}} : \sqrt[7]{x^4 \cdot x^{\frac{1}{2}}} = \sqrt[5]{x^{\frac{10}{7}}} \cdot \sqrt{x^{\frac{5}{7}}} : \sqrt[7]{x^{\frac{9}{2}}} =$$

$$= x^{\frac{2}{7}} \cdot x^{\frac{5}{14}} : x^{\frac{9}{14}} = x^{\frac{2}{7} + \frac{5}{14} - \frac{9}{14}} = x^0 = 1$$

$$21. \sqrt[3]{\frac{a-b}{b^3}} \sqrt{\frac{a+b}{ab}} : \left(\sqrt[3]{\frac{a+b}{a^3}} \sqrt{\frac{a-b}{ab}} : \sqrt{\frac{a+b}{a-b}} \right) = \sqrt[3]{\frac{a-b}{b^3}} \cdot \frac{(a+b)^{\frac{1}{2}}}{a^{\frac{1}{2}} b^{\frac{1}{2}}} : \left(\sqrt[3]{\frac{a+b}{a^3}} \cdot \frac{(a-b)^{\frac{1}{2}}}{a^{\frac{1}{2}} b^{\frac{1}{2}}} : \sqrt{\frac{a+b}{a-b}} \right) =$$

$$= \frac{(a-b)^{\frac{1}{3}} (a+b)^{\frac{1}{6}}}{a^{\frac{1}{6}} b^{\frac{7}{6}}} : \left(\frac{(a+b)^{\frac{1}{3}} (a-b)^{\frac{1}{6}}}{a^{\frac{7}{6}} b^{\frac{1}{6}}} \cdot \frac{(a-b)^{\frac{1}{6}}}{(a+b)^{\frac{1}{6}}} \right) =$$

$$= \frac{(a-b)^{\frac{1}{3}} (a+b)^{\frac{1}{6}}}{a^{\frac{1}{6}} b^{\frac{7}{6}}} \cdot \frac{a^{\frac{7}{6}} b^{\frac{1}{6}}}{(a+b)^{\frac{1}{6}} (a-b)^{\frac{1}{3}}} = \frac{a}{b}$$

Pag. 139

$$22. \sqrt{2x} \sqrt[3]{2x} \cdot \sqrt{\frac{1}{2x}} \sqrt[3]{\frac{1}{2x}} : \sqrt{x} \sqrt[3]{2x} = \sqrt{2x \cdot 2^{\frac{1}{3}} x^{\frac{1}{3}}} \cdot \sqrt{\frac{1}{2x} \cdot \frac{1}{2^{\frac{1}{3}} x^{\frac{1}{3}}}} : \sqrt{x \cdot 2^{\frac{1}{3}} x^{\frac{1}{3}}} =$$

$$= \sqrt{2x \cdot 2^{\frac{1}{3}} x^{\frac{1}{3}} \cdot \frac{1}{2^{\frac{1}{2}} x^{\frac{1}{2}}} \cdot \frac{1}{2^{\frac{1}{6}} x^{\frac{1}{6}}}} : \sqrt[4]{2^{\frac{1}{3}} x^{\frac{4}{3}}} = \sqrt{2^{\frac{2}{3}} x^{\frac{2}{3}}} : \left(2^{\frac{1}{12}} x^{\frac{1}{3}} \right) = \frac{2^{\frac{1}{3}} x^{\frac{1}{3}}}{2^{\frac{1}{12}} x^{\frac{1}{3}}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

$$23. \sqrt{2a} \sqrt[5]{2a} \cdot \sqrt[3]{2b} \sqrt{2b} : \left(\sqrt[5]{\sqrt{2^3 a^2 b}} \cdot \sqrt[3]{2ab} \sqrt[5]{ab} \right) =$$

$$= \sqrt{2a \cdot 2^{\frac{1}{5}} a^{\frac{1}{5}}} \cdot \sqrt[3]{2b \cdot 2^{\frac{1}{2}} b^{\frac{1}{2}}} : \left(\sqrt[5]{2^{\frac{3}{2}} ab^{\frac{1}{2}}} \cdot \sqrt[3]{2ab} \sqrt[5]{a^{\frac{1}{5}} b^{\frac{1}{5}}} \right) =$$

$$= \sqrt{2^{\frac{6}{5}} a^{\frac{6}{5}}} \cdot \sqrt[3]{2^{\frac{3}{2}} b^{\frac{3}{2}}} : \left(2^{\frac{3}{10}} a^{\frac{1}{5}} b^{\frac{1}{10}} \cdot 2^{\frac{1}{3}} a^{\frac{2}{5}} b^{\frac{2}{5}} \right) =$$

$$= 2^{\frac{3}{5}} a^{\frac{3}{5}} \cdot 2^{\frac{1}{2}} b^{\frac{1}{2}} : \left(2^{\frac{19}{30}} a^{\frac{3}{5}} b^{\frac{1}{2}} \right) = 2^{\frac{3}{5} + \frac{1}{2} - \frac{19}{30}} = 2^{\frac{7}{15}} = \sqrt[15]{2^7}$$

$$24. \sqrt[3]{(a+b)^2} \sqrt{(a+b)^3} \sqrt{a+b} \cdot \sqrt{(a+b)} \sqrt{a+b} =$$

$$\begin{aligned}
 &= \sqrt[3]{(a+b)^2} \sqrt{(a+b)^3 (a+b)^{\frac{1}{2}}} \cdot \sqrt{(a+b)(a+b)^{\frac{1}{2}}} = \\
 &= \sqrt[3]{(a+b)^2 (a+b)^{\frac{7}{4}}} \cdot \sqrt{(a+b)^{\frac{3}{2}}} = \sqrt[3]{(a+b)^{\frac{15}{4}}} \cdot (a+b)^{\frac{3}{4}} = (a+b)^{\frac{5}{4}} \cdot (a+b)^{\frac{3}{4}} = (a+b)^2
 \end{aligned}$$

$$\begin{aligned}
 25. \quad &\left(\sqrt[3]{2} \sqrt[3]{2} \sqrt{\frac{1}{2}} \cdot \sqrt[3]{\frac{1}{2}} \sqrt[3]{2} \sqrt[3]{\frac{1}{4}} \right)^6 \cdot \sqrt[3]{2^2} \sqrt{2} = \left(\sqrt[3]{2} \sqrt[3]{2} \sqrt{\frac{1}{2}} \cdot \sqrt[3]{\frac{1}{2}} \sqrt[3]{2} \sqrt{\frac{1}{2}} \right)^6 \cdot \sqrt[3]{2^2 \cdot 2^2} = \\
 &= \left(\sqrt[3]{2 \cdot 2^{\frac{1}{6}}} \cdot \sqrt[3]{\frac{1}{2} \cdot 2^{\frac{1}{9}}} \right)^6 \cdot \sqrt[3]{2^{\frac{5}{2}}} = \left(2^{\frac{7}{18}} \cdot \sqrt[3]{\frac{1}{2^{\frac{8}{9}}}} \right)^6 \cdot 2^{\frac{5}{6}} = \left(2^{\frac{7}{18}} \cdot \frac{1}{2^{\frac{8}{27}}} \right)^6 \cdot 2^{\frac{5}{6}} = \\
 &= \left(2^{\frac{5}{54}} \right)^6 \cdot 2^{\frac{5}{6}} = 2^{\frac{5}{9}} \cdot 2^{\frac{5}{6}} = 2^{\frac{25}{18}} = 2^{1 + \frac{7}{18}} = 2 \sqrt[18]{2^7}
 \end{aligned}$$

$$\begin{aligned}
 &\sqrt[3]{a} \sqrt{a} \sqrt[4]{a^3} \cdot \sqrt[3]{a} \sqrt{\frac{1}{a}} \sqrt{a} : \sqrt{a} = \sqrt[3]{a} \sqrt{a \cdot a^{\frac{3}{4}}} \cdot \sqrt[3]{a} \sqrt{\frac{1}{a} a^{\frac{1}{2}}} : a^{\frac{1}{2}} = \sqrt[3]{a \cdot a^{\frac{7}{8}}} \cdot \sqrt{a \cdot \frac{1}{a}} : a^{\frac{1}{2}} = \\
 &= \sqrt[3]{a^{\frac{15}{8}}} \cdot \sqrt[3]{a^{\frac{3}{4}}} : a^{\frac{1}{2}} = a^{\frac{5}{8}} \cdot a^{\frac{1}{4}} : a^{\frac{1}{2}} = a^{\frac{5}{8} + \frac{1}{4} - \frac{1}{2}} = a^{\frac{3}{8}} = \sqrt[8]{a^3}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad &\left[\left(\sqrt{a} \sqrt[3]{\frac{1}{a}} \right)^3 : \left(\sqrt[6]{a^3} \sqrt{a} \right)^3 + 2a \sqrt[4]{a^{-7}} \right]^2 \cdot \frac{a \sqrt{a}}{9} = \\
 &= \left[\left(\sqrt{a \cdot \frac{1}{a^{\frac{1}{3}}}} \right)^3 : \left(\sqrt[6]{a^3 \cdot a^{\frac{1}{2}}} \right)^3 + 2a \cdot \frac{1}{a^{\frac{7}{4}}} \right]^2 \cdot \frac{a \cdot a^{\frac{1}{2}}}{9} = \left[\left(\sqrt{a^{\frac{2}{3}}} \right)^3 : \left(\sqrt[6]{a^{\frac{7}{2}}} \right)^3 + \frac{2}{a^{\frac{3}{4}}} \right]^2 \cdot \frac{a^{\frac{3}{2}}}{9} = \\
 &= \left[\left(a^{\frac{1}{3}} \right)^3 : \left(a^{\frac{7}{12}} \right)^3 + \frac{2}{a^{\frac{3}{4}}} \right]^2 \cdot \frac{a^{\frac{3}{2}}}{9} = \left[a : a^{\frac{7}{4}} + \frac{2}{a^{\frac{3}{4}}} \right]^2 \cdot \frac{a^{\frac{3}{2}}}{9} = \left[\frac{1}{a^{\frac{3}{4}}} + \frac{2}{a^{\frac{3}{4}}} \right]^2 \cdot \frac{a^{\frac{3}{2}}}{9} = \\
 &= \left(\frac{3}{a^{\frac{3}{4}}} \right)^2 \cdot \frac{a^{\frac{3}{2}}}{9} = \frac{9}{a^{\frac{3}{2}}} \cdot \frac{a^{\frac{3}{2}}}{9} = 1
 \end{aligned}$$

$$\begin{aligned}
 28. \quad &\sqrt[3]{(a^2+b)} \sqrt{a^2+b} \cdot \sqrt[3]{a^2} \sqrt[5]{a^4} : \sqrt{a^2+b} = \sqrt[3]{(a^2+b)^{\frac{3}{2}}} \cdot \sqrt[3]{a^2 \cdot a^{\frac{4}{5}}} : (a^2+b)^{\frac{1}{2}} = \\
 &= \frac{(a^2+b)^{\frac{1}{2}} \cdot \sqrt[3]{a^{\frac{14}{5}}}}{(a^2+b)^{\frac{1}{2}}} = \sqrt[3]{a^{\frac{14}{5}}} = a^{\frac{14}{15}} = \sqrt[15]{a^{14}}
 \end{aligned}$$

32. V
 33. V
 34. F
 35. F
 36. F
 37. V
 38. V
 39. V