ESERCIZI ASSEGNATI PER LE VACANZE NATALIZIE

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18. $$\frac{\left[-\frac{5}{8}\left(-4+\frac{1}{2}\right)(-7)^{-2}+(-2)^{-4}+\frac{3}{4}(+7)^{-1}\right]\left(-6-\frac{2}{3}\right)}{4^{-1}\cdot\left(\frac{1}{2}-\frac{1}{3}\right):(-5)^{-1}:\left[\left(-\frac{3}{2}\right)^{2}+(-2)^{-3}\right]:\left(2+\frac{3}{7}\right)^{-1}}=$$

$$=\frac{\left[-\frac{5}{8}\left(-\frac{7}{2}\right)\left(\frac{1}{7^2}\right)+\frac{1}{16}+\frac{3}{4}\cdot\frac{1}{7}\right]\left(-\frac{20}{3}\right)}{\frac{1}{4}\cdot\left(\frac{1}{6}\right)\cdot(-5):\left[\frac{9}{4}-\frac{1}{8}\right]\cdot\left(\frac{17}{7}\right)}=$$

$$=\frac{\left[+\frac{5}{7\cdot 16}+\frac{1}{16}+\frac{3}{28}\right]\left(-\frac{20}{3}\right)}{\frac{1}{24}\cdot(-5)\cdot\left(\frac{8}{17}\right)\cdot\left(\frac{17}{7}\right)}=\frac{\frac{24}{16\cdot 7}\cdot\left(-\frac{20}{3}\right)}{-\frac{10}{42}}=\frac{10}{7}\cdot\frac{42}{10}=6$$

19. $$\frac{[(-2)^2-(-2)^{-1}]\left[-2-\left(-\frac{5}{2}\right)^{-1}\right]}{\frac{3}{(-2)^2}\cdot(-2)^{-2}\cdot(-2)^3}-(-5)^{-1}=$$

$$=\frac{\left(4+\frac{1}{2}\right)\left[-2+\frac{2}{5}\right]}{\frac{3}{-2}}+\frac{1}{5}=\frac{9}{2}\cdot\left(-\frac{8}{5}\right)\cdot\left(-\frac{2}{3}\right)+\frac{1}{5}=\frac{25}{5}=5$$

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28. $$\frac{2^{-1}+\frac{3}{2^2}\left(1-\frac{1}{3}\right)-2^{-2}:\left(1-\frac{1}{4}\right)}{(-3)^{-3}\,(-3)^2:(-3)^{-1}-\frac{2}{2^2+1}\,(2+2^{-1})}=\frac{\frac{1}{2}+\frac{3}{4}\left(\frac{2}{3}\right)-\frac{1}{4}\cdot\left(\frac{4}{3}\right)}{1-\frac{2}{5}\cdot\frac{5}{2}}=\frac{1-\frac{1}{3}}{0}$$

$$\frac{-5^2\cdot\frac{-2^3\cdot 2^{-2}}{3-(-2)^3:(-2)^2}}{(3^2+1)^2\cdot 10^{-1}-(2^2+1)\,2^{-3}\,(-2)^4}=\frac{-25\cdot\frac{-2}{3+2}}{10^2\cdot 10^{-1}-5\cdot 2}=\frac{10}{0}$$

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26. $$\frac{(a^{2x+3y})^{2x+y}:(a^{2x-y})^{x+y}}{(a^{x-y})^{2x-y}}=\frac{a^{4x^2+8xy+3y^2}:a^{2x^2+xy-y^2}}{a^{2x^2-3xy+y^2}}=\frac{a^{2x^2+7xy+4y^2}}{a^{2x^2-3xy+y^2}}=a^{10xy+3y^2}$$

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29. $(1+2a^n)^2-(3a^n-1)^2-(-2a^n)^2+1=$

$=1+4a^n+4a^{2n}-9a^{2n}+6a^n-1-4a^{2n}+1=\ -9a^{2n}+10a^n+1$

30. $(x^n - y^n)^2 - 3x^n\,(y^n - x^n) - (2x^n + y^n)^2 =$
$= x^{2n} - 2x^n y^n + y^{2n} - 3x^n y^n + 3\,x^{2n} - 4x^{2n} - 4x^n y^n - y^{2n} = -9x^n y^n$

31. $[(x - 3xy)(x + 3xy)(x^2 - 1)(x^2 + 1) - 9x^2 y^2] : (-x^2) =$
$= [(x^2 - 9x^2 y^2)(x^4 - 1) - 9x^2 y^2] : (-x^2) = (x^6 - x^2 - 9x^6 y^2 + 9x^2 y^2 - 9x^2 y^2) : (-x^2) =$
$= -x^4 + 1 + 9x^4 y^2$

32. $[(3a^2 + 5ab)(3a^2 - 5ab) - 8a^3 b] : (-3a^2) =$
$= (9a^4 - 25a^2 b^2 - 8a^3 b) : (-3a^2) = -3a^2 + \dfrac{25}{3} b^2 + \dfrac{8}{3} ab$

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33. $[(a^n - 1)(a^n + 1)]^2 - (a^{2n} + 2)^2 + 3(2a^{2n} + 1) =$
$= (a^{2n} - 1)^2 - (a^{4n} + 4a^{2n} + 4) + 6a^{2n} + 3 =$
$= a^{4n} - 2a^{2n} + 1 - a^{4n} - 4a^{2n} - 4 + 6a^{2n} + 3 = 0$

34. $(3 - x^n)(1 - x^n) - (2 - x^n)^2 + (1 + x^n)(1 - x^n) =$
$= 3 - 3x^n - x^n + x^{2n} - 4 + 4x^n - x^{2n} + 1 - x^{2n} = -x^{2n}$

45. $$\left\{\left[\frac{(a^{n-2})^{2n-3} : (a^{-n})^{-n+3} : (a^n)^n}{a^{1-4n}} - \frac{2}{3} a^5 - 0{,}5\, a^{n-1}\right]^2 - \frac{1}{9}(a^5)^2\right\} : (-2^2 a^n) =$$
$$= \left\{\left[\frac{a^{2n^2-7n+6} : a^{-n^2-3n} : a^{n^2}}{a^{1-4n}} - \frac{2}{3} a^5 - \frac{1}{2} a^{n-1}\right]^2 - \frac{1}{9} a^{10}\right\} : (-4a^n) =$$
$$= \left\{\left[\frac{a^{-4n+6}}{a^{1-4n}} - \frac{2}{3} a^5 - \frac{1}{2} a^{n-1}\right]^2 - \frac{1}{9} a^{10}\right\} : (-4a^n) =$$
$$= \left\{\left[a^5 - \frac{2}{3} a^5 - \frac{1}{2} a^{n-1}\right]^2 - \frac{1}{9} a^{10}\right\} : (-4a^n) =$$
$$= \left\{\left[\frac{1}{3} a^5 - \frac{1}{2} a^{n-1}\right]^2 - \frac{1}{9} a^{10}\right\} : (-4a^n) =$$
$$= \left\{\frac{1}{9} a^{10} - \frac{1}{3} a^{n+4} + \frac{1}{4} a^{2n-2} - \frac{1}{9} a^{10}\right\} : (-4a^n) =$$
$$= \left\{-\frac{1}{3} a^{n+4} + \frac{1}{4} a^{2n-2}\right\} : (-4a^n) = \frac{1}{12} a^4 - \frac{1}{16} a^{n-2}$$

46. $$\left[\frac{(a^n)^{2n-1} : (a^{n-1})^{n+1} \cdot (a^{n+1})^{n+1}}{(a^{n+2})^{n-1}} + a^{n^2-4}\right]^2 - \frac{1}{2^{-1}} a^{2n^2} - a^{2n^2} \cdot \frac{a^{16} + 1}{a^8} =$$
$$= \left[\frac{a^{2n^2-n} : a^{n^2-1} \cdot a^{n^2+2n+1}}{a^{n^2+n-2}} + a^{n^2-4}\right]^2 - 2a^{2n^2} - a^{2n^2} \cdot \left(a^8 + \frac{1}{a^8}\right) =$$

$$= \left[\frac{a^{2n^2+n+2}}{a^{n^2+n-2}} + a^{n^2-4} \right]^2 - 2a^{2n^2} - a^{2n^2+8} - a^{2n^2-8} =$$

$$= (a^{4+n^2} + a^{n^2-4})^2 - 2a^{2n^2} - a^{2n^2+8} - a^{2n^2-8} =$$

$$= a^{8+2n^2} + a^{2n^2-8} + 2a^{2n^2} - 2a^{2n^2} - a^{2n^2+8} - a^{2n^2-8} = 0$$

47. $$\left[\frac{1}{3} a^{2n-3} - \frac{a^{2n-1}}{a^{n-2}} - 2\,(a^{n-1})^3 : (a^2)^{n-2} \right]^3 =$$

$$= \left[\frac{1}{3} a^{2n-3} - a^{n+1} - 2a^{3n-3} : a^{2n-4} \right]^3 =$$

$$= \left[\frac{1}{3} a^{2n-3} - a^{n+1} - 2a^{n+1} \right]^3 =$$

$$= \left[\frac{1}{3} a^{2n-3} - 3a^{n+1} \right]^3 = \frac{1}{27} a^{6n-9} - a^{5n-5} + 9\,a^{4n-1} - 27a^{3n+3}$$

48. $$\left\{ \left[\frac{(a^{n-2})^{n^2-1} : (a^n)^{n^2-1} \cdot (a^{2n})^n}{a^{2-n}} - 2 \right]^4 - 16 - a^{3n}\,(a^n - 8) \right\} : (8a^n) =$$

$$= \left\{ \left[\frac{a^{n^3-2n^2-n+2} : a^{n^3-n} \cdot a^{2n^2}}{a^{2-n}} - 2 \right]^4 - 16 - a^{4n} + 8\,a^{3n} \right\} : (8a^n) =$$

$$= \left\{ \left[\frac{a^2}{a^{2-n}} - 2 \right]^4 - 16 - a^{4n} + 8\,a^{3n} \right\} : (8a^n) =$$

$$= \left\{ \left[a^n - 2 \right]^4 - 16 - a^{4n} + 8\,a^{3n} \right\} : (8a^n) =$$

$$= (a^{4n} - 8a^{3n} + 24a^{2n} - 32a^n + 16 - 16 - a^{4n} + 8a^{3n}) : (8a^n) =$$

$$= (+24a^{2n} - 32a^n) : (8a^n) = 3a^n - 4$$

49. $$\left\{ a^n + a^{7n} + \left[0{,}5\,a^{2n-1} : (-2a^{n-2}) - \frac{3}{4} a^{n+1} \right]^7 : (-0{,}\bar{1}\,a^7) : 9 \right\}^4 =$$

$$= \left\{ a^n + a^{7n} + \left[\frac{1}{2} a^{2n-1} : (-2a^{n-2}) - \frac{3}{4} a^{n+1} \right]^7 : \left(-\frac{1}{9} a^7 \right) : 9 \right\}^4 =$$

$$= \left\{ a^n + a^{7n} + \left[-\frac{1}{4} a^{n+1} - \frac{3}{4} a^{n+1} \right]^7 : \left(-\frac{1}{9} a^7 \right) : 9 \right\}^4 =$$

$$= \left\{ a^n + a^{7n} - a^{7n+7} : \left(-\frac{1}{9} a^7 \right) : 9 \right\}^4 =$$

$$= (a^n + a^{7n} + a^{7n})^4 = (a^n + 2a^{7n})^4 = a^{4n} + 8a^{10n} + 24a^{16n} + 32a^{22n} + 16a^{28n}$$

50. $\{[(2^{80} - 2^{79})^2 + 2^{157}] : 2^{150} - 2^7\} : 2^8 = 1$

$\{[2^{160} - 2^{160} + 2^{158} + 2^{157}] : 2^{150} - 2^7\} : 2^8 = 1$

$\{+ 2^8 + 2^7 - 2^7\} : 2^8 = 1$

$\{+ 2^8\} : 2^8 = 1 \qquad 1 = 1$

51. $(2^6 + 2^5 + 2^4)^2 - (2^6 - 2^5)(2^5 + 2^6) - 2^8 = 9 \cdot 2^{10}$

$2^{12} + 2^{10} + 2^8 + 2^{12} + 2^{11} + 2^{10} - 2^{12} + 2^{10} - 2^8 = 9 \cdot 2^{10}$

$2^{12} + 2^{10} + 2^{11} + 2^{10} + 2^{10} = 9 \cdot 2^{10}$

$4 \cdot 2^{10} + 2^{10} + 2 \cdot 2^{10} + 2^{10} + 2^{10} = 9 \cdot 2^{10} \qquad 9 \cdot 2^{10} = 9 \cdot 2^{10}$

52. $\dfrac{(3^{10} + 3^{11})^2}{(3^6 - 3^5)^3} = 2 \cdot 3^5 \qquad \dfrac{3^{20} + 3^{22} + 2 \cdot 3^{21}}{3^{18} - 3^{18} + 3^{17} - 3^{15}} = 2 \cdot 3^5$

$\dfrac{3^{20} + 9 \cdot 3^{20} + 6 \cdot 3^{20}}{9 \cdot 3^{15} - 3^{15}} = 2 \cdot 3^5 \qquad \dfrac{16 \cdot 3^{20}}{8 \cdot 3^{15}} = 2 \cdot 3^5$

$\dfrac{(2^6 + 2^7)^2}{(2^5 + 2^6 - 2^4)^2} = \left(\dfrac{12}{5}\right)^2 \qquad \dfrac{[2^6(1+2)]^2}{[2^4(2+2^2-1)]^2} = \left(\dfrac{12}{5}\right)^2$

$\dfrac{2^{12} \cdot 3^2}{2^8 \cdot 5^2} = \left(\dfrac{2^2 \cdot 3}{5}\right)^2 \qquad \dfrac{2^4 \cdot 3^2}{5^2} = \dfrac{2^4 \cdot 3^2}{5^2}$

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53. $(a+b)^3 + (2a+b)^3 = (3a+2b)[a(2a+b) + (a+b)^2]$

$a^3 + b^3 + 3a^2b + 3ab^2 + 8a^3 + b^3 + 12a^2b + 6ab^2 = (3a+2b)[a(2a+b) + (a+b)^2]$

$9a^3 + 2b^3 + 15a^2b + 9ab^2 = (3a+2b)[a(2a+b) + (a+b)^2]$

$9a^3 + 2b^3 + 15a^2b + 9ab^2 = (3a+2b)[2a^2 + ab + a^2 + b^2 + 2ab]$

$9a^3 + 2b^3 + 15a^2b + 9ab^2 = (3a+2b)[3a^2 + 3ab + b^2]$

$9a^3 + 2b^3 + 15a^2b + 9ab^2 = 9a^3 + 9a^2b + 3ab^2 + 6a^2b + 6ab^2 + 2b^3$

$9a^3 + 2b^3 + 15a^2b + 9ab^2 = 9a^3 + 2b^3 + 15a^2b + 9ab^2$

60. $\left(\dfrac{3}{2}x^2 - 1\right)^3 - 3\left(\dfrac{3}{2}x^2 - 1\right)^2 + \left(\dfrac{3}{2}x^2 - 1\right)^2\left(7 - \dfrac{3}{2}x^2 + 1\right) =$

$= \dfrac{27}{8}x^6 - \dfrac{27}{4}x^4 + \dfrac{9}{2}x^2 - 1 - 3\left(\dfrac{9}{4}x^4 - 3x^2 + 1\right) + \left(\dfrac{9}{4}x^4 - 3x^2 + 1\right)\left(8 - \dfrac{3}{2}x^2\right) =$

$= \dfrac{27}{8}x^6 - \dfrac{27}{4}x^4 + \dfrac{9}{2}x^2 - 1 - \dfrac{27}{4}x^4 + 9x^2 - 3 + 18x^4 - 24x^2 + 8 - \dfrac{27}{8}x^6 + \dfrac{9}{2}x^4 - \dfrac{3}{2}x^2 =$

$= -12x^2 + 4 + 9x^4$

oppure, molto più semplicemente: $A^3 - 3A^2 + A^2(7 - A) = A^3 - 3A^2 + 7A^2 - A^3 = 4A^2$

$4\left(\dfrac{3}{2}x^2 - 1\right)^2 = 4\left(\dfrac{9}{4}x^4 - 3x^2 + 1\right) = 9x^4 - 12x^2 + 4$