

1. $\log_6 (3x - 1) > 0$

c.a.: $3x - 1 > 0$

$$\log_6 (3x - 1) > 0 \Rightarrow \log_6 (3x - 1) > \log_6 1 \Rightarrow 3x - 1 > 1$$

$$\begin{cases} 3x - 1 > 0 \\ 3x - 1 > 1 \end{cases} \Rightarrow 3x - 1 > 1 \Rightarrow 3x > 2 \Rightarrow x > \frac{2}{3}$$

2. $\log_3 (2x - 4) > \log_3 (5 - x)$

c.a.: $\begin{cases} 2x - 4 > 0 \\ 5 - x > 0 \end{cases} \Rightarrow \begin{cases} x > 2 \\ x < 5 \end{cases} \Rightarrow 2 < x < 5$

$$\log_3 (2x - 4) > \log_3 (5 - x) \Rightarrow 2x - 4 > 5 - x \Rightarrow x > 3$$

$$\begin{cases} 2 < x < 5 \\ x > 3 \end{cases} \Rightarrow 3 < x < 5$$

3. $\log_5 2x + \log_5 \frac{5}{2}x \geq 1$

c.a.: $\begin{cases} 2x > 0 \\ \frac{5}{2}x > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > 0 \end{cases} \Rightarrow x > 0$

$$\log_5 2x + \log_5 \frac{5}{2}x \geq 1 \Rightarrow \log_5 \left(2x \frac{5}{2}x \right) \geq \log_5 5 \Rightarrow 5x^2 \geq 5$$

$$x^2 \geq 1 \Rightarrow x \leq -1 \vee x \geq 1$$

$$\begin{cases} x \leq -1 \vee x \geq 1 \\ x > 0 \end{cases} \Rightarrow x \geq 1$$

$$4. \quad \log_{\frac{2}{3}} 3x + \log_{\frac{2}{3}} (x+2) \geq \log_{\frac{2}{3}} \frac{7}{3}$$

$$c.a.: \begin{cases} 3x > 0 \\ x+2 > 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x > -2 \end{cases} \Rightarrow x > 0$$

$$\log_{\frac{2}{3}} (3x^2 + 6x) \geq \log_{\frac{2}{3}} \frac{7}{3} \Rightarrow 3x^2 + 6x \leq \frac{7}{3}$$

$$9x^2 + 18x - 7 \leq 0 \quad x_{1,2} = \frac{-9 \pm \sqrt{81 + 63}}{9} = \frac{-9 \pm 12}{9} = \begin{cases} -\frac{7}{3} \\ \frac{1}{3} \end{cases}$$

$$\begin{cases} -\frac{7}{3} \leq x \leq \frac{1}{3} \\ x > 0 \end{cases}$$

$$0 < x \leq \frac{1}{3}$$

$$5. \quad \ln(x+1) + \ln(3x+1) \geq \ln 8$$

$$c.a.: \begin{cases} x+1 > 0 \\ 3x+1 > 0 \end{cases} \Rightarrow \begin{cases} x > -1 \\ x > -\frac{1}{3} \end{cases} \Rightarrow x > -\frac{1}{3}$$

$$\ln(x+1)(3x+1) \geq \ln 8 \Rightarrow 3x^2 + x + 3x + 1 \geq 8$$

$$3x^2 + 4x - 7 \geq 0 \Rightarrow x_{1,2} = \frac{-2 \pm \sqrt{4 + 21}}{3} = \begin{cases} 1 \\ -\frac{7}{3} \end{cases}$$

$$\begin{cases} x \leq -\frac{7}{3} \vee x \geq 1 \\ x > -\frac{1}{3} \end{cases}$$

$$x \geq 1$$