

$$1. \quad 8^{3x-4} = 8^{\frac{5+18x^2}{6x+7}}$$

$$3x - 4 = \frac{5 + 18x^2}{6x + 7} \Rightarrow 18x^2 + 21x - 24x - 28 = 5 + 18x^2$$

$$c.a.: x \neq -\frac{7}{6}$$

$$-3x = 33 \Rightarrow$$

$$x = -11$$

$$2. \quad 2^{2x+2} - 9 \cdot 2^x + 2 = 0$$

$$(2^x)^2 \cdot 2^2 - 9 \cdot 2^x + 2 = 0$$

$$\text{Pongo: } 2^x = t \quad 4t^2 - 9t + 2 = 0$$

$$t_{1,2} = \frac{9 \pm \sqrt{81 - 32}}{8} = \left\langle \begin{array}{l} 2 \\ \frac{1}{4} \end{array} \right.$$

$$t = 2 \Rightarrow 2^x = 2 \Rightarrow 2^x = 2^1 \Rightarrow x = 1$$

$$t = \frac{1}{4} \Rightarrow 2^x = \left(\frac{1}{2}\right)^2 \Rightarrow 2^x = 2^{-2} \Rightarrow x = -2$$

$$3. \quad 3^{2x+2} - 82 \cdot 3^x + 9 \leq 0$$

$$(3^x)^2 \cdot 3^2 - 82 \cdot 3^x + 9 \leq 0$$

$$\text{Pongo: } 3^x = t \Rightarrow 9t^2 - 82t + 9 \leq 0 \Rightarrow t_{1,2} = \frac{41 \pm \sqrt{1681 - 81}}{9} = \left\langle \begin{array}{l} 9 \\ \frac{1}{9} \end{array} \right.$$

$$\frac{1}{9} \leq t \leq 9 \Rightarrow \frac{1}{9} \leq 3^x \leq 9 \Rightarrow 3^{-2} \leq 3^x \leq 3^2 \Rightarrow -2 \leq x \leq 2$$

$$4. \quad \ln 7x + \ln \frac{x}{7} = 0$$

$$c.a.: x > 0: \quad \ln \left(7x \cdot \frac{x}{7} \right) = \ln 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \Rightarrow x = 1$$

$$5. \quad \log_4(x+4) + \log_4(x-1) = \log_4 14$$

$$c.a.: \begin{cases} x+4 > 0 \\ x-1 > 0 \end{cases} \Rightarrow \begin{cases} x > -4 \\ x > 1 \end{cases} \Rightarrow x > 1$$

$$\log_4(x+4)(x-1) = \log_4 14$$

$$x^2 + 4x - x - 4 = 14 \Rightarrow x^2 + 3x - 18 = 0 \Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{9 + 72}}{2} = \begin{cases} 3 \\ -6 \end{cases}$$

$$x = -6 \quad \text{non acc. per c.a.}$$

$$x = 3$$

$$6. \quad \log_{\frac{1}{5}} \frac{x+4}{x} < 1$$

$$\begin{cases} \frac{x+4}{x} > 0 \\ \frac{x+4}{x} > \left(\frac{1}{5}\right)^1 \end{cases} \Rightarrow \frac{x+4}{x} > \left(\frac{1}{5}\right)^1$$

$$\frac{5(x+4) - x}{5x} > 0$$

$$\frac{4x+20}{5x} > 0$$

$$N > 0: \quad x > -5$$

$$D > 0: \quad x > 0$$

$$x < -5 \vee x > 0$$

$$7. \quad \log_{\frac{3}{5}}(7x+1) < \log_{\frac{3}{5}}(9x-1)$$

$$c.a.: \begin{cases} 7x+1 > 0 \\ 9x-1 > 0 \end{cases} \Rightarrow \begin{cases} x > -\frac{1}{7} \\ x > \frac{1}{9} \end{cases} \Rightarrow x > \frac{1}{9}$$

$$7x+1 > 9x-1 \Rightarrow -2x > -2$$

$$\begin{cases} x > \frac{1}{9} \\ x < 1 \end{cases} \Rightarrow \frac{1}{9} < x < 1$$