

$$1. \quad 16^{\frac{5x-3}{x+1}} = 16^{\frac{5-x}{3x+1}}$$

$$\frac{5x-3}{x+1} = \frac{5-x}{3x+1} \Rightarrow 15x^2 + 5x - 9x - 3 = 5x + 5 - x^2 - x$$

$$c.a.: x \neq -1; x \neq -\frac{1}{3}$$

$$16x^2 - 8x - 8 = 0 \Rightarrow 2x^2 - x - 1 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \left\langle \begin{array}{l} 1 \\ -\frac{1}{2} \end{array} \right.$$

$$x = 1; \quad x = -\frac{1}{2}$$

$$2. \quad 2^{2x+3} - 17 \cdot 2^x + 2 = 0$$

$$(2^x)^2 \cdot 2^3 - 17 \cdot 2^x + 2 = 0$$

$$\text{Pongo: } 2^x = t \quad 8t^2 - 17t + 2 = 0$$

$$t_{1,2} = \frac{17 \pm \sqrt{289 - 64}}{16} = \left\langle \begin{array}{l} 2 \\ \frac{1}{8} \end{array} \right.$$

$$t = 2 \Rightarrow 2^x = 2 \Rightarrow 2^x = 2^1 \Rightarrow x = 1$$

$$t = \frac{1}{8} \Rightarrow 2^x = \left(\frac{1}{2}\right)^3 \Rightarrow 2^x = 2^{-3} \Rightarrow x = -3$$

$$3. \quad 3^{2x+1} - 28 \cdot 3^x + 9 \leq 0$$

$$(3^x)^2 \cdot 3^1 - 28 \cdot 3^x + 9 \leq 0$$

$$\text{Pongo: } 3^x = t \Rightarrow 3t^2 - 28t + 9 \leq 0 \Rightarrow t_{1,2} = \frac{14 \pm \sqrt{196 - 27}}{3} = \left\langle \begin{array}{l} 9 \\ \frac{1}{3} \end{array} \right.$$

$$\frac{1}{3} \leq t \leq 9 \Rightarrow \frac{1}{3} \leq 3^x \leq 9 \Rightarrow 3^{-1} \leq 3^x \leq 3^2 \Rightarrow -1 \leq x \leq 2$$

$$4. \quad \ln 8x + \ln \frac{x}{2} = 0$$

$$c.a.: x > 0: \quad \ln \left( 8x \cdot \frac{x}{2} \right) = \ln 1 \Rightarrow 4x^2 = 1 \Rightarrow x = \pm \frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$5. \quad \log_4(x-5) + \log_4(x-7) = \log_4 3$$

$$c.a.: \begin{cases} x-5 > 0 \\ x-7 > 0 \end{cases} \Rightarrow \begin{cases} x > 5 \\ x > 7 \end{cases} \Rightarrow x > 7$$

$$\log_4(x-5)(x-7) = \log_4 3$$

$$x^2 - 7x - 5x + 35 = 3 \Rightarrow x^2 - 12x + 32 = 0 \Rightarrow x_{1,2} = 6 \pm \sqrt{36 - 32} = \begin{cases} 8 \\ 4 \end{cases}$$

$$x = 4 \quad \text{non acc. per c.a.}$$

$$x = 8$$

$$6. \quad \log_3 \frac{x+4}{x} < 1$$

$$\begin{cases} \frac{x+4}{x} > 0 \\ \frac{x+4}{x} < 3^1 \end{cases} \Rightarrow$$

$$\begin{cases} x < -4 \vee x > 0 \\ \frac{4-2x}{x} < 0 \end{cases} \Rightarrow$$

$$\begin{cases} x < -4 \vee x > 0 \\ x < 0 \vee x > 2 \end{cases} \Rightarrow$$

$$x < -4 \vee x > 2$$

$$7. \quad \log_{\frac{1}{2}}(3x-5) < \log_{\frac{1}{2}}(2x-1)$$

$$c.a.: \begin{cases} 3x-5 > 0 \\ 2x-1 > 0 \end{cases} \Rightarrow \begin{cases} x > \frac{5}{3} \\ x > \frac{1}{2} \end{cases} \Rightarrow x > \frac{5}{3}$$

$$\log_{\frac{1}{2}}(3x-5) < \log_{\frac{1}{2}}(2x-1)$$

$$3x-5 > 2x-1 \Rightarrow x > 4 \quad \begin{cases} x > \frac{5}{3} \\ x > 4 \end{cases} \Rightarrow x > 4$$