1. $27^{2x} + \frac{2}{3} \cdot 3^{3x+1} = 3$

$3^{6x} + \frac{2}{3} \cdot 3^{3x} \cdot 3 - 3 = 0$ Pongo: $3^{3x} = t$

$t^2 + 2t - 3 = 0$ $t_{1,2} = \frac{-1 \pm \sqrt{1+3}}{1} \begin{cases} 1 \\ -3 \end{cases}$

$t = 1 \Rightarrow 3^{3x} = 1 \Rightarrow 3^{3x} = 3^0 \Rightarrow 3x = 0 \Rightarrow$ $x = 0$

$t = -3 \Rightarrow 3^{3x} = -3 \Rightarrow$ *imp.*

2. $25^x - 5^{x+1} < 0$

$5^{2x} - 5^x \cdot 5 < 0$ $5^x (5^x - 5) < 0$ $5^x - 5 < 0$

$5^x < 5$ $5^x < 5^1$ $x < 1$

3. $\log_{3/4}(2x-3) + \log_{3/4}(x+2) = \log_{3/4}(x-4)$

$c.a.: \begin{cases} 2x - 3 > 0 \\ x + 2 > 0 \\ x - 4 > 0 \end{cases}$ $\begin{cases} x > \frac{3}{2} \\ x > -2 \\ x > 4 \end{cases}$ $x > 4$

$\log_{3/4}[(2x-3)(x+2)] = \log_{3/4}(x-4)$

$2x^2 + 4x - 3x - 6 = x - 4$ $2x^2 - 2 = 0$

$2x^2 = 2$ $x = \pm 1$ non accettabili per le c.a. $\Rightarrow$ *imp.*

4. $\log_2(x^2 - 4) - \log_2(x-2) > 1$

$c.a.: \begin{cases} x^2 - 4 > 0 \\ x - 2 > 0 \end{cases}$ $\begin{cases} x < -2 \ \vee \ x > 2 \\ x > 2 \end{cases}$ $c.a.: x > 2$

$\log_2[(x-2)(x+2)] - \log_2(x-2) > 1$

$\log_2(x-2) + \log_2(x+2) - \log_2(x-2) > 1$

$\log_2(x+2) > 1$ $x + 2 > 2$ $x > 0$

$\begin{cases} x > 2 \\ x > 0 \end{cases}$ $x > 2$

5. $sen\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{6}+x\right)$

$= sen\,\frac{\pi}{3}\cos\,x+\cos\,\frac{\pi}{3}\,sen\,x+\cos\,\frac{\pi}{6}\cos\,x-sen\,\frac{\pi}{6}\,sen\,x=$

$=\frac{\sqrt{3}}{2}\cos\,x+\frac{1}{2}\,sen\,x+\frac{\sqrt{3}}{2}\cos\,x-\frac{1}{2}\,sen\,x=\sqrt{3}\cos\,x$

6. $\frac{1-\cos\,2\alpha}{1+\cos\,2\alpha}\cdot ctg\,\alpha$

$=\frac{1-(1-2\,sen^2\,\alpha)}{1+2\cos^2\alpha-1}\cdot\frac{\cos\,\alpha}{sen\,\alpha}=\frac{1-1+2\,sen^2\,\alpha}{2\cos^2\alpha}\cdot\frac{\cos\,\alpha}{sen\,\alpha}=\frac{2\,sen^2\,\alpha}{2\cos^2\alpha}\cdot\frac{\cos\,\alpha}{sen\,\alpha}=tg\,\alpha$

7. $sen\,\alpha\cos\,2\alpha-\cos\,\alpha\,sen\,2\alpha=\cos\left(\frac{\pi}{2}+\alpha\right)$

$sen\,\alpha\,(2\cos^2\alpha-1)-\cos\,\alpha\,(2\,sen\,\alpha\cos\alpha)=-sen\,\alpha$

$2\cos^2\alpha\,sen\,\alpha-sen\,\alpha-2\cos^2\alpha\,sen\,\alpha=-sen\,\alpha$ $\quad -sen\,\alpha=-sen\,\alpha$

8. $sen\,2\alpha\,tg\,\alpha+\cos^2\alpha=2-\cos\,2\alpha-sen^2\,\alpha$

$2\,sen\,\alpha\cos\,\alpha\,\frac{sen\,\alpha}{\cos\,\alpha}+\cos^2\alpha=2-(\cos^2\alpha-sen^2\,\alpha)-sen^2\,\alpha$

$2\,sen^2\,\alpha+\cos^2\alpha=2-\cos^2\alpha+sen^2\,\alpha-sen^2\,\alpha$

$2\,(1-\cos^2\alpha)+\cos^2\alpha=2-\cos^2\alpha$

$2-2\cos^2\alpha+\cos^2\alpha=2-\cos^2\alpha$ $\quad 2-\cos^2\alpha=2-\cos^2\alpha$