Semplifica le seguenti espressioni:

1. $\sqrt{9-4\sqrt{2}}\cdot\sqrt{9+4\sqrt{2}}-\dfrac{5}{\sqrt{7}-\sqrt{2}}+\sqrt{2}\left(1-\dfrac{7}{2}\sqrt{2}\right)$

$=\sqrt{81-32}\ -\dfrac{5}{\sqrt{7}-\sqrt{2}}\cdot\dfrac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}+\sqrt{2}-7=$

$=\sqrt{49}\ -\dfrac{5(\sqrt{7}+\sqrt{2})}{5}+\sqrt{2}-7=7-\sqrt{7}-\sqrt{2}+\sqrt{2}-7=\ -\sqrt{7}$

2. $\dfrac{\sqrt{3}\,(2\sqrt{3}-\sqrt{15}+\sqrt{21})-(\sqrt{108}+\sqrt{21}):\sqrt{3}}{\sqrt{7}\,(\sqrt{8}+\sqrt{5})-\sqrt{5}\,(\sqrt{18}+\sqrt{7})}$

$=\dfrac{6-3\sqrt{5}+3\sqrt{7}-\sqrt{36}-\sqrt{7}}{\sqrt{7}\,(2\sqrt{2}+\sqrt{5})-\sqrt{5}\,(3\sqrt{2}+\sqrt{7})}=$

$=\dfrac{6-3\sqrt{5}+3\sqrt{7}-6-\sqrt{7}}{2\sqrt{14}+\sqrt{35}-3\sqrt{10}-\sqrt{35}}=\dfrac{2\sqrt{7}-3\sqrt{5}}{2\sqrt{14}-3\sqrt{10}}=\dfrac{2\sqrt{7}-3\sqrt{5}}{\sqrt{2}\,(2\sqrt{7}-3\sqrt{5})}=\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\ \dfrac{\sqrt{2}}{2}$

3. $\sqrt[3]{(a-b)\sqrt{a-b}}\cdot\sqrt{2\sqrt[5]{2}}:\sqrt{a-b}$

$=\sqrt[3]{\sqrt{(a-b)^3}}\cdot\sqrt{\sqrt[5]{2^5\cdot 2}}:\sqrt{a-b}=\ \sqrt{a-b}\cdot\sqrt[10]{2^6}:\sqrt{a-b}=\sqrt[10]{2^6}\ =\ \sqrt[5]{8}$

4. $(\sqrt{6}+\sqrt{3})^2-(\sqrt{2}+\sqrt{8})^2+9$

$=6+3+6\sqrt{2}-(\sqrt{2}+2\sqrt{2})^2+9=9+6\sqrt{2}-(3\sqrt{2})^2+9=18+6\sqrt{2}-18=\ 6\sqrt{2}$

5. $\left(\sqrt{5}-\dfrac{1}{\sqrt{5}}\right)^2+\dfrac{2}{\sqrt{5}}-\left(2+\dfrac{1}{\sqrt{5}}\right):\sqrt{5}$

$=5+\dfrac{1}{5}-2+\dfrac{2}{\sqrt{5}}-\dfrac{2}{\sqrt{5}}-\dfrac{1}{5}=\ 3$

6. $\dfrac{13}{3\sqrt{2}-\sqrt{5}} - 3\sqrt{2} + \left(\dfrac{\sqrt{5}}{2} - 1\right)^2 + \left(\sqrt{5} - \sqrt{\dfrac{125}{16}}\right) : \sqrt{5}$

$= \dfrac{13}{3\sqrt{2}-\sqrt{5}} \cdot \dfrac{3\sqrt{2}+\sqrt{5}}{3\sqrt{2}+\sqrt{5}} - 3\sqrt{2} + \dfrac{5}{4} + 1 - \sqrt{5} + \left(\sqrt{5} - \dfrac{5}{4}\sqrt{5}\right) : \sqrt{5} =$

$= \dfrac{13(3\sqrt{2}+\sqrt{5})}{18-5} - 3\sqrt{2} + \dfrac{5}{4} + 1 - \sqrt{5} + 1 - \dfrac{5}{4} = 3\sqrt{2} + \sqrt{5} - 3\sqrt{2} + 1 - \sqrt{5} + 1 = \boxed{2}$

Risolvi:

7. $\sqrt{3}(x-\sqrt{2}) + (x-\sqrt{2})(x-\sqrt{3}) = x^2$

$x\sqrt{3} - \sqrt{6} + x^2 - x\sqrt{3} - x\sqrt{2} + \sqrt{6} = x^2$

$-x\sqrt{2} = 0$ $\qquad$ $x = 0$

8. $\dfrac{x-1}{x+\sqrt{5}} - \dfrac{2x+3}{x-\sqrt{5}} - \dfrac{x^2-\sqrt{5}-4}{5-x^2} = 0$

$\dfrac{x-1}{x+\sqrt{5}} - \dfrac{2x+3}{x-\sqrt{5}} - \dfrac{x^2-\sqrt{5}-4}{(\sqrt{5}-x)(\sqrt{5}+x)} = 0$

$\dfrac{(x-1)(x-\sqrt{5}) - (2x+3)(x+\sqrt{5}) + x^2 - \sqrt{5} - 4}{(x+\sqrt{5})(\sqrt{5}+x)} = 0$ $\qquad$ $c.a.: x \neq \pm\sqrt{5}$

$x^2 - x\sqrt{5} - x + \sqrt{5} - 2x^2 - 2x\sqrt{5} - 3x - 3\sqrt{5} + x^2 - \sqrt{5} - 4 = 0$

$-3x\sqrt{5} - 4x = 3\sqrt{5} + 4$ $\qquad$ $-x(3\sqrt{5}+4) = 3\sqrt{5} + 4$ $\qquad$ $x = -1$

9. $\begin{cases} x\sqrt{2} - y\sqrt{3} = \sqrt{2} - 1 \\ (\sqrt{2} - 1)\, x = 4 - y\sqrt{3} \end{cases}$

$$\begin{cases} x\sqrt{2} - y\sqrt{3} = \sqrt{2} - 1 \\ (\sqrt{2} - 1)\, x + y\sqrt{3} = 4 \end{cases}$$

$$\overline{x\sqrt{2} + x\sqrt{2} - x = 3 + \sqrt{2}}$$

$$x\,(2\sqrt{2} - 1) = 3 + \sqrt{2}$$

$$\frac{x\,(2\sqrt{2} - 1)}{2\sqrt{2} - 1} = \frac{3 + \sqrt{2}}{2\sqrt{2} - 1} \cdot \frac{2\sqrt{2} + 1}{2\sqrt{2} + 1}$$

$$x = \frac{6\sqrt{2} + 3 + 4 + \sqrt{2} + \sqrt{2}}{8 - 1} = \frac{7\sqrt{2} + 7}{7} = \frac{7\,(\sqrt{2} + 1)}{7} = \sqrt{2} + 1$$

Sostituisco il valore di x determinato nella seconda equazione:

$$(\sqrt{2} - 1)\,(\sqrt{2} + 1) + y\sqrt{3} = 4$$

$$2 - 1 + y\sqrt{3} = 4 \qquad y\sqrt{3} = 3 \qquad y = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$$

$$\begin{cases} x = \sqrt{2} + 1 \\ y = \sqrt{3} \end{cases}$$

10. $(\sqrt{2} - 8)\,(x + 4) + 3x\sqrt{2} \leq -4\sqrt{2}\,(2\sqrt{2} + 1)$

$$x\sqrt{2} + 4\sqrt{2} - 8x - 32 + 3x\sqrt{2} \leq -16 - 4\sqrt{2}$$

$$-8x + 4x\sqrt{2} \leq 16 - 8\sqrt{2}$$

$$4x\,(-2 + \sqrt{2}) \leq -8\,(-2 + \sqrt{2})$$

$$\frac{4x\,(-2 + \sqrt{2})}{-2 + \sqrt{2}} \leq \frac{-8\,(-2 + \sqrt{2})}{-2 + \sqrt{2}} \qquad 4x \geq -8 \qquad x \geq -2$$