1. $4\,sen^2 x \cos^2 x = 1$

$(2\,sen\,x\,\cos x)^2 = 1$ $\quad sen^2\,2x = 1$ $\quad sen\,2x = \pm 1$

$2x = \frac{\pi}{2} + k\pi$ $\quad x = \frac{\pi}{4} + k\frac{\pi}{2}$

2. $\cos x - ctg\,x = 0$

$\cos x - \frac{\cos x}{sen\,x} = 0$ $\quad \frac{\cos x}{sen\,x}(sen\,x - 1) = 0$ $\quad ctg\,x\,(sen\,x - 1) = 0$

$ctg\,x = 0$ $\quad x = \frac{\pi}{2} + k\pi$

$sen\,x = 1$ $\quad x = \frac{\pi}{2} + 2k\pi$

$x = \frac{\pi}{2} + k\pi$

3. $3\,sen\,x + 2\,(sen^2 x + 2\cos^2 x) = 2$

$3\,sen\,x + 2\,(sen^2 x + 2\,(1 - sen^2 x)) = 2$

$3\,sen\,x + 2\,(sen^2 x + 2 - 2\,sen^2 x) = 2$

$3\,sen\,x + 2\,(2 - sen^2 x) = 2$ $\quad 3\,sen\,x + 4 - 2\,sen^2 x = 2$

$2\,sen^2 x - 3\,sen\,x - 2 = 0$ $\quad sen\,x = \frac{3 \pm \sqrt{9+16}}{4} = \left\langle \begin{matrix} 2 \\ -\frac{1}{2} \end{matrix} \right.$

$sen\,x = 2$ $\quad$ *imp.*

$sen\,x = -\frac{1}{2}$ $\quad x = \frac{7}{6}\pi + 2k\pi$ $\quad x = \frac{11}{6}\pi + 2k\pi$

4. $\cos x\,(2\cos x + 1) = 1$

$2\cos^2 x + \cos x - 1 = 0$ $\quad \cos x = \frac{-1 \pm \sqrt{1+8}}{4} = \left\langle \begin{matrix} \frac{1}{2} \\ -1 \end{matrix} \right.$

$\cos x = \frac{1}{2}$ $\quad x = \frac{\pi}{3} + 2k\pi$ $\quad x = \frac{5}{3}\pi + 2k\pi$

$\cos x = -1$ $\quad x = \pi + 2k\pi$

5. $(2\,ctg\,x + \sqrt{3})^2 = 3$

$2\,ctg\,x + \sqrt{3} = \pm\sqrt{3}$

$2\,ctg\,x + \sqrt{3} = \sqrt{3}$ $\qquad ctg\,x = 0$ $\qquad x = \frac{\pi}{2} + k\pi$

$2\,ctg\,x + \sqrt{3} = -\sqrt{3}$ $\qquad ctg\,x = -\sqrt{3}$ $\qquad x = \frac{5}{6}\pi + k\pi$

6. $\cos\left(x - \frac{2}{3}\pi\right) = \cos(\pi + x)$

$\cos x \cos\frac{2}{3}\pi + sen\,x\; sen\,\frac{2}{3}\pi = -\cos x$

$-\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\,sen\,x = -\cos x$ $\qquad \cos x + \sqrt{3}\,sen\,x = 0$

È un'equazione omogenea di primo grado, divido per cos x, sapendo che cos x = 0 non è soluzione dell'equazione:

$1 + \sqrt{3}\,tg\,x = 0$ $\qquad tg\,x = -\frac{\sqrt{3}}{3}$ $\qquad x = -\frac{\pi}{6} + k\pi$

7. $\frac{\cos x}{1 + sen\,x} = \sqrt{3}$ $\qquad c.a.: sen\,x \neq -1$

$\cos x = \sqrt{3}\,(1 + sen\,x)$ $\qquad \cos x - \sqrt{3} - \sqrt{3}\,sen\,x = 0$

Si tratta di un'equazione lineare, che risolvo in modo grafico, ponendo cos x = X e sen x = Y:

$\begin{cases} X - \sqrt{3}\,Y = \sqrt{3} \\ X^2 + Y^2 = 1 \end{cases}$ $\qquad \begin{cases} X = \sqrt{3}\,Y + \sqrt{3} \\ 3Y^2 + 3 + 6Y + Y^2 = 1 \end{cases}$ $\qquad \begin{cases} X = -\sqrt{3}\,Y + \sqrt{3} \\ 2Y^2 + 3Y + 1 = 0 \end{cases}$

$Y = \frac{-3 \pm \sqrt{9 - 8}}{4} = \left\langle \begin{array}{l} -1 \quad non\ acc.\ per\ c.a. \\ -\frac{1}{2} \end{array} \right.$ $\qquad \begin{cases} X = -\frac{\sqrt{3}}{2} + \sqrt{3} \\ Y = -\frac{1}{2} \end{cases}$ $\qquad \begin{cases} X = \frac{\sqrt{3}}{2} \\ Y = -\frac{1}{2} \end{cases}$

$x = \frac{11}{6}\pi + 2k\pi$

8. $5\, sen^2 x + (1-\sqrt{3})\, sen\,x \cos x + (4-\sqrt{3}) \cos^2 x = 4$

È un'equazione di secondo grado riconducibile a omogenea, moltiplicando 4 per $\cos^2 x + sen^2 x$:

$$5\, sen^2 x + (1-\sqrt{3})\, sen\,x \cos x + 4\cos^2 x - \sqrt{3}\cos^2 x = 4(\cos^2 x + sen^2 x)$$

$$5\, sen^2 x + (1-\sqrt{3})\, sen\,x \cos x + 4\cos^2 x - \sqrt{3}\cos^2 x = 4\cos^2 x + 4\, sen^2 x$$

$$sen^2 x + (1-\sqrt{3})\, sen\,x \cos x - \sqrt{3}\cos^2 x = 0$$

Divido per $\cos^2 x$, avendo verificato che $x = \frac{\pi}{2} + k\pi$ non è soluzione dell'equazione

$$tg^2 x + (1-\sqrt{3})\, tg\,x - \sqrt{3} = 0$$

$$tg\,x = \frac{\sqrt{3}-1 \pm \sqrt{1+3-2\sqrt{3}+4\sqrt{3}}}{2} = \frac{\sqrt{3}-1 \pm (1+\sqrt{3})}{2} = \left\langle \begin{array}{l} -1 \\ \sqrt{3} \end{array} \right.$$

$tg\,x = -1$ $\qquad$ $x = \frac{3}{4}\pi + k\pi$

$tg\,x = \sqrt{3}$ $\qquad$ $x = \frac{\pi}{3} + k\pi$

9. $sen\,3x - \cos 3x = 1$

È un'equazione lineare che posso risolvere in forma grafica, ponendo cos 3x = X e sen 3x = Y:

$$\begin{cases} Y - X = 1 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = Y - 1 \\ Y^2 - 2Y + 1 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = Y - 1 \\ 2Y^2 - 2Y = 0 \end{cases}$$

$$2Y(Y-1) = 0 \qquad \left\langle \begin{array}{l} Y = 0 \\ Y = 1 \end{array} \right.$$

$\begin{cases} X = -1 \\ Y = 0 \end{cases}$ $\qquad 3x = \pi + 2k\pi$ $\qquad x = \frac{\pi}{3} + \frac{2}{3}k\pi$

$\begin{cases} X = 0 \\ Y = 1 \end{cases}$ $\qquad 3x = \frac{\pi}{2} + 2k\pi$ $\qquad x = \frac{\pi}{6} + \frac{2}{3}k\pi$