1. Applicando opportunamente le formule di addizione e sottrazione, calcola la seguente funzione goniometrica:

$$\cos\frac{17}{12}\pi = \cos\left(2\pi - \frac{7}{12}\pi\right) = \cos\frac{7}{12}\pi = \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) =$$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - sen\frac{\pi}{4}\,sen\frac{\pi}{3} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}\,(1-\sqrt{3})}{4}$$

2. Sapendo che: $sen\,\alpha = \frac{3}{4}$, con $0 < \alpha < \frac{\pi}{2}$, calcola $sen\,2\alpha$.

Determiniamo innanzi tutto $\cos\alpha = \sqrt{1 - sen^2\alpha} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$$sen\,2\alpha = 2\,sen\,\alpha\cos\alpha = 2\cdot\frac{\sqrt{7}}{4}\cdot\frac{3}{4} = \frac{3\sqrt{7}}{8}$$

3. Sapendo che: $sen\,\alpha = \frac{4}{5}$, con $0 < \alpha < \frac{\pi}{2}$, calcola seno e coseno di $\frac{\alpha}{2}$.

Determiniamo innanzi tutto $\cos\alpha = \sqrt{1 - sen^2\alpha} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

$$sen\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5}$$

Semplifica le seguenti espressioni:

4. $sen\left(\frac{\pi}{3} - x\right) + \cos\left(\frac{\pi}{6} - x\right)$

$$= sen\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\,sen\,x + \cos\frac{\pi}{6}\cos x + sen\frac{\pi}{6}\,sen\,x =$$

$$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\,sen\,x + \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\,sen\,x = \sqrt{3}\cos x$$

5. $\cos 2\alpha - sen\,2\alpha\,ctg\,\alpha$

$$= 2\cos^2\alpha - 1 - 2\,sen\,\alpha\cos\alpha\,\frac{\cos\alpha}{sen\,\alpha} =$$

$$= 2\cos^2\alpha - 1 - 2\cos^2\alpha = -1$$

6. $\dfrac{1+\cos 2\alpha}{1-\cos 2\alpha}\cdot tg\ \alpha$

$$= \frac{1+2\cos^2\alpha-1}{1-(1-2\,sen^2\alpha)}\cdot\frac{sen\ \alpha}{\cos\alpha} = \frac{2\cos^2\alpha}{1-1+2\,sen^2\alpha}\cdot\frac{sen\ \alpha}{\cos\alpha} = \frac{2\cos^2\alpha}{2\,sen^2\alpha}\cdot\frac{sen\ \alpha}{\cos\alpha} = ctg\ \alpha$$

7. $tg\ \dfrac{\alpha}{2} - 2\,\dfrac{sen^2\dfrac{\alpha}{2}}{sen\ \alpha}$

$$= \frac{1-\cos\alpha}{sen\ \alpha} - 2\left(\frac{1-\cos\alpha}{2}\right)\cdot\frac{1}{sen\ \alpha} = \frac{1-\cos\alpha-1+\cos\alpha}{sen\ \alpha} = 0$$

Verifica le seguenti identità:

8. $\cos\alpha\ sen\ 2\alpha - sen\ \alpha\cos 2\alpha = \cos\left(\dfrac{\pi}{2}-\alpha\right)$

$\cos\alpha\,(2\,sen\,\alpha\cos\alpha) - sen\ \alpha\,(\cos^2\alpha - sen^2\alpha) = sen\,\alpha$

$2\cos^2\alpha\ sen\,\alpha - \cos^2\alpha\ sen\ \alpha + sen^3\alpha = sen\,\alpha$

$\cos^2\alpha\ sen\ \alpha + sen^3\alpha = sen\,\alpha$

$sen\ \alpha\,(\cos^2\alpha + sen^2\alpha) = sen\,\alpha$ $\qquad sen\ \alpha = sen\ \alpha$

9. $3+\cos 2\alpha = 2+2\cos^2\alpha$

$3+2\cos^2\alpha - 1 = 2+2\cos^2\alpha$ $\qquad 2+2\cos^2\alpha = 2+2\cos^2\alpha$

10. $sen\ 2\alpha\ tg\ \alpha + \cos^2\alpha = 2-\cos 2\alpha - sen^2\alpha$

$2\ sen\ \alpha\cos\alpha\,\dfrac{sen\ \alpha}{\cos\alpha} + \cos^2\alpha = 2-(\cos^2\alpha - sen^2\alpha) - sen^2\alpha$

$2\ sen^2\alpha + \cos^2\alpha = 2-\cos^2\alpha + sen^2\alpha - sen^2\alpha$

$2\,(1-\cos^2\alpha) + \cos^2\alpha = 2-\cos^2\alpha$

$2-2\cos^2\alpha + \cos^2\alpha = 2-\cos^2\alpha$ $\qquad 2-\cos^2\alpha = 2-\cos^2\alpha$

11. $sen^2\,\alpha - 2\cos\alpha\; sen^2\,\dfrac{\alpha}{2} = sen\,\alpha\; tg\,\dfrac{\alpha}{2}$

$$sen^2\,\alpha - 2\cos\alpha\left(\frac{1-\cos\alpha}{2}\right) = sen\,\alpha\,\frac{1-\cos\alpha}{sen\,\alpha}$$

$$sen^2\,\alpha - \cos\alpha + \cos^2\alpha = 1-\cos\alpha \qquad\qquad 1-\cos\alpha = 1-\cos\alpha$$

12. $\dfrac{2\cos^2\dfrac{\alpha}{2}\,(1-\cos\alpha)}{2\cos^2\dfrac{\alpha}{2}\cdot sen\,\alpha} = \dfrac{sen\,\alpha}{1+\cos\alpha}$

$$\frac{1-\cos\alpha}{sen\,\alpha} = \frac{sen\,\alpha}{1+\cos\alpha}$$

$$\frac{1-\cos^2\alpha}{sen\,\alpha\,(1+\cos\alpha)} = \frac{sen^2\,\alpha}{sen\,\alpha\,(1+\cos\alpha)} \qquad\qquad sen^2\,\alpha = sen^2\,\alpha$$

13. Trasforma in $t = tg\,\dfrac{\alpha}{2}$ la seguente espressione:

$$\frac{sen\,\alpha + 2\cos\alpha + 2}{sen\,\alpha}$$

$$\frac{sen\,\alpha + 2\cos\alpha + 2}{sen\,\alpha} = \frac{\dfrac{2t}{1+t^2} + 2\,\dfrac{1-t^2}{1+t^2} + 2}{\dfrac{2t}{1+t^2}} =$$

$$= \frac{2t + 2 - 2t^2 + 2 + 2t^2}{2t} = \frac{2t+4}{2t} = \frac{2\,(t+2)}{2t} = \frac{t+2}{t}$$