1. Applicando opportunamente le formule di addizione e sottrazione, calcola la seguente funzione goniometrica:

$$sen\,\frac{17}{12}\pi = sen\left(2\pi - \frac{7}{12}\pi\right) = -sen\,\frac{7}{12}\pi = -sen\left(\frac{\pi}{4}+\frac{\pi}{3}\right) =$$

$$= -\left(sen\,\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\,sen\,\frac{\pi}{3}\right) = -\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}\,(1+\sqrt{3})}{4}$$

2. Sapendo che: $\cos\alpha = \frac{3}{4}$, con $0 < \alpha < \frac{\pi}{2}$, calcola $sen\,2\alpha$.

Determiniamo innanzi tutto $sen\,\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$$sen\,2\alpha = 2\,sen\,\alpha\cos\alpha = 2\cdot\frac{3}{4}\cdot\frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{8}$$

3. Sapendo che: $sen\,\alpha = \frac{3}{5}$, con $0 < \alpha < \frac{\pi}{2}$, calcola seno e coseno di $\frac{\alpha}{2}$.

Determiniamo innanzi tutto $\cos\alpha = \sqrt{1-sen^2\alpha} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$$sen\,\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

Semplifica le seguenti espressioni:

4. $sen\left(\frac{\pi}{3}+x\right) + \cos\left(\frac{\pi}{6}+x\right)$

$$= sen\,\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\,sen\,x + \cos\frac{\pi}{6}\cos x - sen\,\frac{\pi}{6}\,sen\,x =$$

$$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\,sen\,x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\,sen\,x = \sqrt{3}\cos x$$

5. $\cos 2\alpha + sen\,2\alpha\,tg\,\alpha$

$$= \cos^2\alpha - sen^2\alpha + 2\,sen\,\alpha\cos\alpha\,\frac{sen\,\alpha}{\cos\alpha} =$$

$$= \cos^2\alpha - sen^2\alpha + 2\,sen^2\alpha = \cos^2\alpha + sen^2\alpha = 1$$

6. $\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha} \cdot ctg\ \alpha$

$$= \frac{1-(1-2\,sen^2\alpha)}{1+2\cos^2\alpha-1} \cdot \frac{\cos\alpha}{sen\ \alpha} = \frac{1-1+2\,sen^2\alpha}{2\cos^2\alpha} \cdot \frac{\cos\alpha}{sen\ \alpha} = \frac{2\,sen^2\alpha}{2\cos^2\alpha} \cdot \frac{\cos\alpha}{sen\ \alpha} = \boxed{tg\ \alpha}$$

7. $tg\ \dfrac{\alpha}{2} + 2\,\dfrac{\cos^2\dfrac{\alpha}{2}}{sen\ \alpha}$

$$= \frac{1-\cos\alpha}{sen\ \alpha} + 2\left(\frac{1+\cos\alpha}{2}\right) \cdot \frac{1}{sen\ \alpha} = \frac{1-\cos\alpha+1+\cos\alpha}{sen\ \alpha} = \boxed{\frac{2}{sen\ \alpha}}$$

Verifica le seguenti identità:

8. $sen\ \alpha \cos 2\alpha - \cos\alpha\ sen\ 2\alpha = \cos\left(\dfrac{\pi}{2}+\alpha\right)$

$sen\ \alpha\,(\cos^2\alpha - sen^2\alpha) - \cos\alpha\,(2\,sen\,\alpha\,\cos\alpha) = -sen\,\alpha$

$\cos^2\alpha\ sen\ \alpha - sen^3\alpha - 2\cos^2\alpha\ sen\,\alpha = -sen\,\alpha$

$-\cos^2\alpha\ sen\ \alpha - sen^3\alpha = -sen\,\alpha$

$-\,sen\ \alpha\,(\cos^2\alpha + sen^2\alpha) = -sen\,\alpha$ $\qquad\boxed{-sen\ \alpha = -sen\ \alpha}$

9. $1+\cos 2\alpha = 2 - 2\,sen^2\alpha$

$1+1-2\,sen^2\alpha = 2-2\,sen^2\alpha$ $\qquad\boxed{2-2\,sen^2\alpha = 2-2\,sen^2\alpha}$

10. $sen\ 2\alpha\ tg\ \alpha + \cos^2\alpha = 2 - \cos 2\alpha - sen^2\alpha$

$2\ sen\ \alpha\cos\alpha\,\dfrac{sen\ \alpha}{\cos\alpha} + \cos^2\alpha = 2 - (\cos^2\alpha - sen^2\alpha) - sen^2\alpha$

$2\ sen^2\alpha + \cos^2\alpha = 2 - \cos^2\alpha + sen^2\alpha - sen^2\alpha$

$2\,(1-\cos^2\alpha) + \cos^2\alpha = 2 - \cos^2\alpha$

$2 - 2\cos^2\alpha + \cos^2\alpha = 2 - \cos^2\alpha$ $\qquad\boxed{2-\cos^2\alpha = 2-\cos^2\alpha}$

11. $sen\ \alpha\ tg\ \frac{\alpha}{2} = sen^2\ \alpha - 2\cos\alpha\ sen^2\ \frac{\alpha}{2}$

$$sen\ \alpha\ \frac{1-\cos\alpha}{sen\ \alpha} = sen^2\ \alpha - 2\cos\alpha\left(\frac{1-\cos\ \alpha}{2}\right)$$

$$1-\cos\alpha = sen^2\ \alpha - \cos\alpha + \cos^2\ \alpha$$

$$1-\cos\alpha = 1-\cos\alpha$$

12. $\frac{2\ sen^2\ \frac{\alpha}{2}\ (1+\cos\alpha)}{2\ sen^2\ \frac{\alpha}{2}\cdot sen\,\alpha} = \frac{sen\,\alpha}{1-\cos\alpha}$

$$\frac{1+\cos\alpha}{sen\,\alpha} = \frac{sen\,\alpha}{1-\cos\alpha}$$

$$\frac{1-\cos^2\alpha}{sen\,\alpha\ (1-\cos\alpha)} = \frac{sen^2\ \alpha}{sen\,\alpha\ (1-\cos\alpha)}$$

$$sen^2\ \alpha = sen^2\ \alpha$$

13. Trasforma in $t = tg\ \frac{\alpha}{2}$ la seguente espressione:

$$\frac{2\ sen\ \alpha + \cos\ \alpha + 1}{sen\ \alpha}$$

$$\frac{2\ sen\ \alpha + \cos\ \alpha + 1}{sen\ \alpha} = \frac{2\ \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} + 1}{\frac{2t}{1+t^2}} =$$

$$= \frac{4t + 1 - t^2 + 1 + t^2}{2t} = \frac{4t+2}{2t} = \frac{2\ (2t+1)}{2t} = \frac{2t+1}{t}$$