Sviluppa mediante le formule di duplicazione, di addizione, di sottrazione e gli archi associati ed eventualmente semplifica:

1. $tg\ 2\alpha - \dfrac{2\cos\alpha}{\cos\alpha - sen\,\alpha} + \dfrac{2\cos^2\alpha}{\cos 2\alpha}$

$$= \frac{sen\,2\alpha}{\cos 2\alpha} - \frac{2\cos\alpha}{\cos\alpha - sen\,\alpha} + \frac{2\cos^2\alpha}{\cos 2\alpha} = \frac{2\,sen\,\alpha\cos\alpha + 2\cos^2\alpha}{\cos 2\alpha} - \frac{2\cos\alpha}{\cos\alpha - sen\,\alpha} =$$

$$= \frac{2\cos\alpha\,(sen\,\alpha + \cos\alpha)}{\cos^2\alpha - sen^2\alpha} - \frac{2\cos\alpha}{\cos\alpha - sen\,\alpha} =$$

$$= \frac{2\cos\alpha\,(sen\,\alpha + \cos\alpha)}{(\cos\alpha - sen\,\alpha)(\cos\alpha + sen\,\alpha)} - \frac{2\cos\alpha}{\cos\alpha - sen\,\alpha} =$$

$$= \frac{2\cos\alpha}{\cos\alpha - sen\,\alpha} - \frac{2\cos\alpha}{\cos\alpha - sen\,\alpha} = \boxed{0}$$

2. $2\,sen^2\left(\alpha + \dfrac{\pi}{4}\right) - sen\,2\alpha$

$$= 2\left(sen\,\alpha\cos\frac{\pi}{4} + \cos\alpha\,sen\,\frac{\pi}{4}\right)^2 - 2\,sen\,\alpha\cos\alpha =$$

$$= 2\left(\frac{\sqrt{2}}{2}sen\,\alpha + \frac{\sqrt{2}}{2}\cos\alpha\right)^2 - 2\,sen\,\alpha\cos\alpha =$$

$$= 2\left(\frac{1}{2}sen^2\alpha + \frac{1}{2}\cos^2\alpha + \cos\alpha\,sen\,\alpha\right) - 2\,sen\,\alpha\cos\alpha =$$

$$= 2\left(\frac{1}{2} + \cos\alpha\,sen\,\alpha\right) - 2\,sen\,\alpha\cos\alpha = 1 + 2\cos\alpha\,sen\,\alpha - 2\cos\alpha\,sen\,\alpha = \boxed{1}$$

3. $$\frac{sen\ (\pi - \alpha) + \cos\ 2\pi + \cos\left(\frac{\pi}{2} + \alpha\right) + \cos\ 2\alpha}{\cos^2\ \alpha}$$

$$= \frac{sen\ \alpha + 1 - sen\ \alpha + \cos^2\ \alpha - sen^2\ \alpha}{\cos^2\ \alpha} = \frac{1 + \cos^2\ \alpha - (1 - \cos^2\ \alpha)}{\cos^2\ \alpha} =$$

$$= \frac{1 + \cos^2\ \alpha - 1 + \cos^2\ \alpha}{\cos^2\ \alpha} = \frac{2\cos^2\ \alpha}{\cos^2\ \alpha} = 2$$

4. $$\cos\left(\alpha - \frac{\pi}{4}\right)\cos\left(\frac{\pi}{4} + \alpha\right) + 3 - \frac{\cos\ 2\alpha}{2}$$

$$\left(\cos\ \alpha\ \cos\frac{\pi}{4} + sen\ \alpha\ sen\frac{\pi}{4}\right)\left(\cos\ \alpha\ \cos\frac{\pi}{4} - sen\ \alpha\ sen\frac{\pi}{4}\right) + 3 - \frac{\cos^2\ \alpha - sen^2\ \alpha}{2} =$$

$$\left(\frac{\sqrt{2}}{2}\cos\ \alpha + \frac{\sqrt{2}}{2}sen\ \alpha\right)\left(\frac{\sqrt{2}}{2}\cos\ \alpha - \frac{\sqrt{2}}{2}sen\ \alpha\right) + 3 - \frac{1}{2}\cos^2\ \alpha + \frac{1}{2}sen^2\ \alpha =$$

$$= \frac{1}{2}\cos^2\ \alpha - \frac{1}{2}sen^2\ \alpha + 3 - \frac{1}{2}\cos^2\ \alpha + \frac{1}{2}sen^2\ \alpha = 3$$

5. $$sen\left(\frac{\pi}{3} + \alpha\right) + \frac{sen\ (\pi + \alpha)}{2} + 4 + \frac{\sqrt{3}}{2}sen\left(\frac{3}{2}\pi + \alpha\right)$$

$$sen\frac{\pi}{3}\cos\ \alpha + sen\ \alpha\ \cos\frac{\pi}{3} + \frac{-sen\ \alpha}{2} + 4 + \frac{\sqrt{3}}{2}(-\cos\ \alpha) =$$

$$= \frac{\sqrt{3}}{2}\cos\ \alpha + \frac{1}{2}sen\ \alpha - \frac{1}{2}sen\ \alpha + 4 - \frac{\sqrt{3}}{2}\cos\ \alpha = 4$$

6. Verifica la seguente identità: $\cos 2\alpha = 2\cos^2\alpha - 1$

Applicando le formule di duplicazione a primo membro: $\cos^2\alpha - sen^2\alpha = 2\cos^2\alpha - 1$

$\cos^2\alpha - (1 - \cos^2\alpha) = 2\cos^2\alpha - 1$

$\cos^2\alpha - 1 + \cos^2\alpha = 2\cos^2\alpha - 1$

$2\cos^2\alpha - 1 = 2\cos^2\alpha - 1$ c.v.d.

7. Calcola $sen\,(2\alpha - \beta)$ sapendo che: $\cos\alpha = \frac{3}{5}\quad 0 < \alpha < \frac{\pi}{2}$ e $sen\,\beta = \frac{5}{13}\quad 0 < \beta < \frac{\pi}{2}$.

$sen\,(2\alpha - \beta) = sen\,2\alpha\cos\beta - \cos 2\alpha\, sen\,\beta =$

$= 2\,sen\,\alpha\cos\alpha\cos\beta - (\cos^2\alpha - sen^2\alpha)\,sen\,\beta =$

E, avendo ricavato che: $\cos\alpha = \frac{3}{5}\quad 0 < \alpha < \frac{\pi}{2} \quad\Rightarrow\quad sen\,\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$sen\,\beta = \frac{5}{13}\quad 0 < \beta < \frac{\pi}{2} \quad\Rightarrow\quad \cos\beta = \sqrt{1 - sen^2\beta} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$, sostituendo i valori:

$= 2\cdot\frac{4}{5}\cdot\frac{3}{5}\cdot\frac{12}{13} - \left(\frac{9}{25} - \frac{16}{25}\right)\cdot\frac{5}{13} = \frac{288}{325} + \frac{35}{325} = \frac{323}{325}$

8. Dimostra la formula del coseno della somma di due angoli, dando per dimostrata la formula del coseno della differenza.

La formula da dimostrare è: $\cos(\alpha + \beta) = \cos\alpha\cos\beta - sen\,\alpha\, sen\,\beta$

Partendo dalla formula di sottrazione del coseno: $\cos(\alpha + \beta) = \cos\alpha\cos\beta + sen\,\alpha\, sen\,\beta$

Posso vedere $\alpha + \beta = \alpha - (-\beta)$, perciò,

$\cos(\alpha + \beta) = \cos(\alpha - (-\beta)) =$ e, applicando la formula di sottrazione del coseno e gli archi associati:

$= \cos\alpha\cos(-\beta) + sen\,\alpha\, sen\,(-\beta) = \cos\alpha\cos\beta - sen\,\alpha\, sen\,\beta$ c.v.d.