Calcola il valore delle seguenti espressioni:

1. $sen\ 15\pi + \cos(-13\pi) - \frac{1}{4}\cos(14\pi) + sen\ \frac{21}{2}\pi$

$= 0 - 1 - \frac{1}{4}\cdot(1) + 1 = -1 - \frac{1}{4} + 1 = -\frac{1}{4}$

2. $\dfrac{\sqrt{3}\cos\frac{11}{6}\pi + sen\left(-\frac{13}{2}\pi\right) + \cos\left(-\frac{11}{3}\pi\right)}{2\ sen\left(-\frac{8}{2}\pi\right) + \cos 6\pi - 4\ sen\ \frac{9}{2}\pi}$

$= \dfrac{\sqrt{3}\cdot\frac{\sqrt{3}}{2} + (-1) + \frac{1}{2}}{2(-1) + 1 - 4\cdot 1} = \dfrac{\frac{3}{2} - 1 + \frac{1}{2}}{-2 + 1 - 4} = -\frac{1}{5}$

3. $\left[sen\left(-\frac{13}{6}\pi\right) - ctg\left(\frac{3}{4}\pi\right) + \sqrt{2}\ sen\left(\frac{9}{4}\pi\right)\right]\left[\sqrt{2}\cos\left(\frac{7}{4}\pi\right) + sen\left(\frac{7}{2}\pi\right)\right]$

$= \left(-\frac{1}{2} - (-1) + \sqrt{2}\cdot\frac{\sqrt{2}}{2}\right)\left(\sqrt{2}\cdot\frac{\sqrt{2}}{2} - 1\right) = \left(-\frac{1}{2} + 1 + 1\right)(1 - 1) = 0$

4. $\left[\frac{5}{\sqrt{3}}ctg\left(\frac{11}{3}\pi\right) + \frac{8}{3}tg\left(\frac{9}{4}\pi\right) - \cos\pi\right]\left[3 + sen\left(\frac{17}{2}\pi\right)\right]$

$= \left[\frac{5}{\sqrt{3}}\left(-\frac{\sqrt{3}}{3}\right) + \frac{8}{3}\cdot 1 + 1\right](3 + 1) = \left(-\frac{5}{3} + \frac{8}{3} + 1\right)\cdot 4 = 8$

Calcola i valori delle rimanenti funzioni goniometriche, essendo dati:

5. $\cos\alpha = -\frac{5}{13} \qquad \pi < \alpha < \frac{3}{2}\pi$

$$sen\ \alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$tg\ \alpha = \frac{-\sqrt{1-\cos^2\alpha}}{\cos\alpha} = -\frac{12}{13}\cdot\left(-\frac{13}{5}\right) = \frac{12}{5}$$

$$ctg\ \alpha = \frac{1}{tg\ \alpha} = \frac{5}{12}$$

6. $tg\ \alpha = \frac{15}{8} \qquad \pi < \alpha < \frac{3}{2}\pi$

$$sen\ \alpha = -\frac{tg\ \alpha}{\sqrt{1+tg^2\alpha}} = -\frac{15}{8}\cdot\frac{1}{\sqrt{1+\frac{225}{64}}} = -\frac{15}{8}\cdot\frac{8}{17} = -\frac{15}{17}$$

$$\cos\alpha = -\frac{1}{\sqrt{1+tg^2\alpha}} = -\frac{1}{\sqrt{1+\frac{225}{64}}} = -\frac{8}{17}$$

$$ctg\ \alpha = \frac{1}{tg\ \alpha} = \frac{8}{15}$$

Trasforma le seguenti espressioni in altre contenenti solo $\cos\alpha$:

7. $ctg\ \alpha\ tg\ \alpha - sen^2\alpha + \cos\alpha \ = 1 - sen^2\alpha + \cos\alpha = \cos^2\alpha + \cos\alpha$

8. $(2 + sen\ \alpha\ ctg\ \alpha)(2 - sen\ \alpha\ ctg\ \alpha) + sen^2\alpha\ ctg^2\alpha - 4\ sen^2\alpha$

$$= 4 - sen^2\alpha\ ctg^2\alpha + sen^2\alpha\ ctg^2\alpha - 4(1-\cos^2\alpha) = 4 - 4 + 4\cos^2\alpha = 4\cos^2\alpha$$

Trasforma le seguenti espressioni in altre contenenti solo $sen\ \alpha$:

9. $\frac{1}{sen^2 \alpha} - ctg^2\ \alpha + 3$

$$= \frac{1}{sen^2\ \alpha} - \frac{\cos^2\ \alpha}{sen^2\ \alpha} + 3 = \frac{1 - \cos^2\ \alpha}{sen^2\ \alpha} + 3 = \frac{sen^2\ \alpha}{sen^2\ \alpha} + 3 = 1 + 3 = \boxed{4}$$

10. $ctg\ \alpha \cos\ \alpha - \frac{1}{sen\ \alpha} + 2\,sen\,\alpha$

$$= \frac{\cos\ \alpha}{sen\ \alpha} \cos\ \alpha - \frac{1}{sen\ \alpha} + 2\,sen\,\alpha = \frac{\cos^2 \alpha - 1}{sen\ \alpha} + 2\,sen\,\alpha =$$

$$= \frac{-sen^2\ \alpha}{sen\,\alpha} + 2\,sen\,\alpha = -sen\,\alpha + 2\,sen\,\alpha = \boxed{sen\ \alpha}$$