

$$1. \quad 3^{4x+1} - 82 \cdot 9^x + 27 = 0$$

$$3 \cdot 3^{4x} - 82 \cdot 3^{2x} + 27 = 0$$

$$\text{Pongo: } 3^{2x} = t$$

$$3t^2 - 82t + 27 = 0$$

$$t_{1,2} = \frac{41 \pm \sqrt{1681 - 81}}{3} \left\langle \begin{array}{l} 27 \\ \frac{1}{3} \end{array} \right.$$

$$t = 27 \quad \Rightarrow \quad 3^{2x} = 27 \quad \Rightarrow \quad 3^{2x} = 3^3 \quad \Rightarrow \quad x = \frac{3}{2}$$

$$t = \frac{1}{3} \quad \Rightarrow \quad 3^{2x} = \frac{1}{3} \quad \Rightarrow \quad 3^{2x} = 3^{-1} \quad \Rightarrow \quad x = -\frac{1}{2}$$

$$2. \quad \frac{2^x - 16}{27 - 3^x} \geq 0$$

$$N \geq 0: 2^x - 16 \geq 0 \quad \Rightarrow \quad 2^x \geq 2^4 \quad \Rightarrow \quad x \geq 4$$

$$D > 0: 27 - 3^x > 0 \quad \Rightarrow \quad 3^x < 3^3 \quad \Rightarrow \quad x < 3$$

Dallo studio dei segni ottengo:

$$3 < x \leq 4$$

$$3. \quad 4^{3x+2} - 65 \cdot 8^x \geq -4$$

$$4^2 \cdot 4^{3x} - 65 \cdot 8^x + 4 \geq 0$$

$$4^2 \cdot 2^{6x} - 65 \cdot 2^{3x} + 4 \geq 0$$

$$\text{Pongo: } 2^{3x} = t$$

$$16t^2 - 65t + 4 \geq 0$$

$$t_{1,2} = \frac{65 \pm \sqrt{65^2 - 256}}{32} \left\langle \begin{array}{l} 4 \\ \frac{1}{16} \end{array} \right.$$

$$t \leq \frac{1}{16} \quad \vee \quad t \geq 4$$

$$2^{3x} \leq \frac{1}{16} \quad \vee \quad 2^{3x} \geq 4 \quad \Rightarrow \quad 3x \leq -4 \quad \vee \quad 3x \geq 2 \quad \Rightarrow \quad x \leq -\frac{4}{3} \quad \vee \quad x \geq \frac{2}{3}$$

$$4. \quad \log_5 (x - 3) + \log_5 (x + 1) = 3 + \log_5 \frac{12}{125}$$

$$c.a.: \begin{cases} x - 3 > 0 \\ x + 1 > 0 \end{cases} \quad \begin{cases} x > 3 \\ x > -1 \end{cases} \quad c.a.: x > 3$$

$$\log_5 (x - 3) + \log_5 (x + 1) = \log_5 125 + \log_5 \frac{12}{125}$$

$$\log_5 (x - 3) (x + 1) = \log_5 125 \cdot \frac{12}{125}$$

$$\log_5 (x^2 - 2x - 3) = \log_5 12$$

$$x^2 - 2x - 3 = 12$$

$$x^2 - 2x - 15 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1 + 15}}{1} = \begin{cases} 5 \\ -3 \text{ non acc. per c.a.} \end{cases}$$

$$\Rightarrow x = 5$$

$$5. \quad \log_{\frac{1}{4}} (5x + 4) > -1$$

$$c.a.: 5x + 4 > 0 \quad \Rightarrow \quad x > -\frac{4}{5}$$

$$5x + 4 < 4$$

$$5x < 0$$

$$x < 0$$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo:

$$-\frac{4}{5} < x < 0$$

$$6. \quad \log (16x - 3) - 2 \log (2x + 3) + \log \left(\frac{x}{4} - 1 \right) \geq 0$$

$$c.a.: \begin{cases} 16x - 3 > 0 \\ 2x + 3 > 0 \\ \frac{x}{4} - 1 > 0 \end{cases} \quad \begin{cases} x > \frac{3}{16} \\ x > -\frac{3}{2} \\ x > 4 \end{cases} \quad c.a.: x > 4$$

$$\log (16x - 3) + \log \left(\frac{x}{4} - 1 \right) \geq 2 \log (2x + 3)$$

$$\log (16x - 3) \left(\frac{x}{4} - 1 \right) \geq \log (2x + 3)^2$$

$$4x^2 - 16x - \frac{3x}{4} + 3 \geq 4x^2 + 12x + 9$$

$$-\frac{115}{4} x \geq 6$$

$$\frac{115}{4} x \leq -6$$

$$x \leq -\frac{24}{115}$$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo:

imp.