1. $\frac{5^{7x-1}}{25^{x+2}} = 4^{5-5x}$

$\frac{5^{7x-1}}{5^{2x+4}} = 4^{5-5x}$ $\quad 5^{7x-1-2x-4} = 4^{5-5x}$ $\quad 5^{5x-5} = \frac{1}{4^{5-5x}}$

$20^{5x-5} = 1$ $\quad 20^{5x-5} = 20^0$ $\quad 5x-5=0$ $\quad x = 1$

2. $4^x - 5 \cdot 2^{x+2} = -64$

$4^x - 5 \cdot 2^2 \cdot 2^x = -64$ $\quad$ Pongo: $2^x = t$

$t^2 - 20\,t + 64 = 0$ $\quad t_{1,2} = \frac{10 \pm \sqrt{100-64}}{1} \left\langle \begin{array}{l} 4 \\ 16 \end{array} \right.$

$t = 4 \quad \Rightarrow \quad 2^x = 4 \quad \Rightarrow \quad 2^x = 2^2 \quad \Rightarrow \quad x = 2$

$t = 16 \quad \Rightarrow \quad 2^x = 16 \quad \Rightarrow \quad 2^x = 2^4 \quad \Rightarrow \quad x = 4$

3. $(2^x - 4)(5^x + 7)(3^x - 9) > 0$

Primo fattore: $2^x - 4 > 0 \quad \Rightarrow \quad 2^x > 2^2 \quad \Rightarrow \quad x > 2$

Secondo fattore: $5^x + 7 > 0 \quad \forall x \in R$

Terzo fattore: $3^x - 9 > 0 \quad \Rightarrow \quad 3^x > 3^2 \quad \Rightarrow \quad x > 2$

Dallo studio dei segni ottengo: $\forall\, x \neq 2$

4. $\frac{2^x - 8}{25 - 5^x} \geq 0$

$N \geq 0$: $2^x - 8 \geq 0 \quad \Rightarrow \quad 2^x \geq 2^3 \quad \Rightarrow \quad x \geq 3$

$D > 0$: $25 - 5^x > 0 \quad \Rightarrow \quad 5^x < 5^2 \quad \Rightarrow \quad x < 2$

Dallo studio dei segni ottengo: $2 < x \leq 3$

5. $5^{6x} - 4 \cdot 5^{3x+1} \geq 125$

$5^{6x} - 4 \cdot 5 \cdot 5^{3x} - 125 \geq 0$ $\quad$ Pongo: $5^{3x} = t$

$t^2 - 20t - 125 \geq 0$ $\quad t_{1,2} = \frac{10 \pm \sqrt{100+125}}{1} \left\langle \begin{array}{l} 25 \\ -5 \end{array} \right.$ $\quad t \leq -5 \ \vee \ t \geq 25$

$5^{3x} \leq -5 \ \vee \ 5^{3x} \geq 25 \quad \Rightarrow \quad 3x \geq 2 \quad \Rightarrow \quad x \geq \frac{2}{3}$

6. $\log_2(x-5)+\log_2(x-2)=4+\log_2\frac{9}{8}$

$c.a.:\begin{cases} x-5>0 \\ x-2>0 \end{cases}$ $\quad\begin{cases} x>5 \\ x>2 \end{cases}$ $\quad c.a.: x>5$

$\log_2(x-5)+\log_2(x-2)=\log_2 16+\log_2\frac{9}{8}$

$\log_2(x-5)(x-2)=\log_2 16\cdot\frac{9}{8}$ $\quad\log_2(x^2-7x+10)=\log_2 18$

$x^2-7x+10=18$ $\quad x^2-7x-8=0$

$x_{1,2}=\frac{7\pm\sqrt{49+32}}{2}=\left\langle\begin{matrix} 8 \\ -1 \ \ non\ acc.\ per\ c.a. \end{matrix}\right.$ $\quad\Rightarrow\quad x=8$

7. $\ln^2(x+1)-8\ln(x+1)+15=0$ $\quad c.a.: x>-1$

Pongo: $\ln(x+1)=t$ $\quad t^2-8t+15=0$

$t_{1,2}=\frac{4\pm\sqrt{16-15}}{1}\left\langle\begin{matrix} 5 \\ 3 \end{matrix}\right.$

$t=5 \quad\Rightarrow\quad \ln(x+1)=5 \quad\Rightarrow\quad x+1=e^5 \quad\Rightarrow\quad x=e^5-1$

$t=3 \quad\Rightarrow\quad \ln(x+1)=3 \quad\Rightarrow\quad x+1=e^3 \quad\Rightarrow\quad x=e^3-1$

8. $\log_{\frac{1}{2}}(3x-1)>-2$

$c.a.: 3x-1>0 \quad\Rightarrow\quad x>\frac{1}{3}$

$3x-1<4$ $\quad 3x<5$ $\quad x<\frac{5}{3}$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $\frac{1}{3}<x<\frac{5}{3}$

9. $\log(3x-1) - 2\log(x-1) + \log\left(\frac{x}{3}-2\right) \geq 0$

$$c.a.: \begin{cases} 3x-1>0 \\ x-1>0 \\ \frac{x}{3}-2>0 \end{cases} \qquad \begin{cases} x>\frac{1}{3} \\ x>1 \\ x>6 \end{cases} \qquad c.a.: x>6$$

$$\log(3x-1) + \log\left(\frac{x}{3}-2\right) \geq 2\log(x-1)$$

$$\log(3x-1)\left(\frac{x}{3}-2\right) \geq \log(x-1)^2$$

$$x^2 - 6x - \frac{x}{3} + 2 \geq x^2 - 2x + 1$$

$$-\frac{13}{3}x \geq -1 \qquad \frac{13}{3}x \leq 1 \qquad x \leq \frac{3}{13}$$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $imp.$

10. $\ln x^3 - \ln x^2 + 7\ln x - 7 \geq 0$ $\qquad c.a.: x>0$

$$3\ln x - 2\ln x + 7\ln x \geq 7$$

$$8\ln x \geq 7 \qquad \ln x \geq \frac{7}{8}$$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $x \geq e^{\frac{7}{8}}$