1. $\dfrac{9^{7x-1}}{3^{x+2}} = \left(\dfrac{1}{2}\right)^{4-13x}$

$\dfrac{3^{14x-2}}{3^{x+2}} = \dfrac{1}{2^{4-13x}}$ $\quad 3^{14x-2-x-2} = \dfrac{1}{2^{4-13x}}$ $\quad 3^{13x-4} = 2^{13x-4}$

$\left(\dfrac{3}{2}\right)^{13x-4} = 1$ $\quad \left(\dfrac{3}{2}\right)^{13x-4} = \left(\dfrac{3}{2}\right)^{0}$ $\quad 13x-4=0$ $\quad x = \dfrac{4}{13}$

2. $64^x - 5 \cdot 8^{x+\frac{2}{3}} = -64$

$64^x - 5 \cdot 2^2 \cdot 8^x = -64$ $\quad$ Pongo: $8^x = t$

$t^2 - 20t + 64 = 0$ $\quad t_{1,2} = \dfrac{10 \pm \sqrt{100-64}}{1} \left\langle \begin{matrix} 4 \\ 16 \end{matrix} \right.$

$t = 4 \quad \Rightarrow \quad 8^x = 4 \quad \Rightarrow \quad 2^{3x} = 2^2 \quad \Rightarrow \quad x = \dfrac{2}{3}$

$t = 16 \quad \Rightarrow \quad 8^x = 16 \quad \Rightarrow \quad 2^{3x} = 2^4 \quad \Rightarrow \quad x = \dfrac{4}{3}$

3. $(2^x - 4)(5^x + 7)(9 - 3^x) \geq 0$

Primo fattore: $2^x - 4 \geq 0 \quad \Rightarrow \quad 2^x \geq 2^2 \quad \Rightarrow \quad x \geq 2$

Secondo fattore: $5^x + 7 \geq 0 \quad \forall x \in R$

Terzo fattore: $9 - 3^x \geq 0 \quad \Rightarrow \quad 3^x \leq 3^2 \quad \Rightarrow \quad x \leq 2$

Dallo studio dei segni ottengo: $x = 2$

4. $\dfrac{8 - 2^x}{5^x - 125} \geq 0$

$N \geq 0:\ 8 - 2^x \geq 0 \quad \Rightarrow \quad 2^x \leq 2^3 \quad \Rightarrow \quad x \leq 3$

$D > 0:\ 5^x - 125 > 0 \quad \Rightarrow \quad 5^x < 5^3 \quad \Rightarrow \quad x > 3$

Dallo studio dei segni ottengo: $\forall x \in R$

5. $5^{4x} - 4 \cdot 5^{2x+1} < 125$

$5^{4x} - 4 \cdot 5 \cdot 5^{2x} - 125 < 0$ Pongo: $5^{2x} = t$

$t^2 - 20t - 125 < 0$ $t_{1,2} = \dfrac{10 \pm \sqrt{100+125}}{1} \left\langle \begin{matrix} 25 \\ -5 \end{matrix} \right.$ $t \le -5 \;\vee\; t \ge 25$

$-5 < 5^{2x} < 25 \quad \Rightarrow \quad 2x < 2 \quad \Rightarrow \quad x < 1$

6. $\log_2(5-x) + \log_2(2-x) = 4 + \log_2 \dfrac{9}{8}$

$c.a.: \begin{cases} 5 - x > 0 \\ 2 - x > 0 \end{cases}$ $\begin{cases} x < 5 \\ x < 2 \end{cases}$ $c.a.: x < 2$

$\log_2(5-x) + \log_2(2-x) = \log_2 16 + \log_2 \dfrac{9}{8}$

$\log_2(5-x)(2-x) = \log_2 16 \cdot \dfrac{9}{8}$ $\log_2(x^2 - 7x + 10) = \log_2 18$

$x^2 - 7x + 10 = 18$ $x^2 - 7x - 8 = 0$

$x_{1,2} = \dfrac{7 \pm \sqrt{49+32}}{2} = \left\langle \begin{matrix} 8 \quad \textit{non acc. per c.a.} \\ -1 \end{matrix} \right.$ $\Rightarrow \quad x = -1$

7. $\ln^2(2x+1) - 5\ln(2x+1) - 6 = 0$ $c.a.: x > -\dfrac{1}{2}$

Pongo: $\ln(2x+1) = t$ $t^2 - 5t - 6 = 0$ $t_{1,2} = \dfrac{5 \pm \sqrt{25+24}}{2} \left\langle \begin{matrix} 6 \\ -1 \end{matrix} \right.$

$t = 6 \quad \Rightarrow \quad \ln(2x+1) = 6 \quad \Rightarrow \quad 2x + 1 = e^6 \quad \Rightarrow \quad x = \dfrac{e^6 - 1}{2}$

$t = -1 \quad \Rightarrow \quad \ln(2x+1) = -1 \quad \Rightarrow \quad 2x + 1 = e^{-1} \quad \Rightarrow \quad x = \dfrac{e^{-1} - 1}{2}$

8. $\log_{\frac{1}{2}}(4x-1) > -3$

$c.a.: 4x-1>0 \quad \Rightarrow \quad x>\frac{1}{4}$

$4x-1<8 \qquad 4x<9 \qquad x<\frac{9}{4}$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $\frac{1}{4}<x<\frac{9}{4}$

9. $\log(4x-1) - 2\log(x-1) + \log\left(\frac{x}{4}-2\right) \geq 0$

$c.a.: \begin{cases} 4x-1>0 \\ x-1>0 \\ \frac{x}{4}-2>0 \end{cases} \qquad \begin{cases} x>\frac{1}{4} \\ x>1 \\ x>8 \end{cases} \qquad c.a.: x>8$

$\log(4x-1) + \log\left(\frac{x}{4}-2\right) \geq 2\log(x-1)$

$\log(4x-1)\left(\frac{x}{4}-2\right) \geq \log(x-1)^2$

$x^2 - 8x - \frac{x}{4} + 2 \geq x^2 - 2x + 1$

$-\frac{25}{4}x \geq -1 \qquad \frac{25}{4}x \leq 1 \qquad x \leq \frac{4}{25}$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: *imp.*

10. $\log x^3 - \log x^2 + 7\log x + 8 \geq 0 \qquad c.a.: x>0$

$3\log x - 2\log x + 7\log x \geq -8$

$8\log x \geq -8 \qquad \log x \geq -1$

Mettendo a sistema il risultato ottenuto con le c.a. ottengo: $x \geq \frac{1}{10}$