

$$1. \quad 81^x + 2 \cdot (9^x)^2 + 4 \cdot 3^{4x-1} = 117$$

$$3^{4x} + 2 \cdot (3^{2x})^2 + 4 \cdot \frac{3^{4x}}{3} = 117$$

$$3^{4x} + 2 \cdot 3^{4x} + \frac{4}{3} \cdot 3^{4x} = 117$$

$$3^{4x} \left(1 + 2 + \frac{4}{3} \right) = 117$$

$$3^{4x} \cdot \frac{13}{3} = 117$$

$$3^{4x} = 117 \cdot \frac{3}{13}$$

$$3^{4x} = 27$$

$$3^{4x} = 3^3$$

$$4x = 3$$

$$x = \frac{3}{4}$$

$$2. \quad 4^{2x} - 2^{2(x+1)} = 32$$

$$2^{4x} - 4 \cdot 2^{2x} - 32 = 0$$

$$\text{Pongo: } 2^{2x} = t$$

$$t^2 - 4t - 32 = 0$$

$$t_{1,2} = \frac{2 \pm \sqrt{4 + 32}}{1} \begin{cases} 8 \\ -4 \end{cases}$$

$$t = 8 \quad \Rightarrow \quad 2^{2x} = 8 \quad \Rightarrow \quad 2^{2x} = 2^3 \quad \Rightarrow \quad 2x = 3 \quad \Rightarrow$$

$$x = \frac{3}{2}$$

$$t = -4 \quad \Rightarrow \quad 2^{2x} = -4 \quad \Rightarrow \quad \text{imp.}$$

$$3. \quad 3^{2x+1} \geq 5^{-2x-1}$$

$$3^{2x+1} \cdot 5^{2x+1} \geq 1$$

$$15^{2x+1} \geq 1$$

$$15^{2x+1} \geq 15^0$$

$$2x+1 \geq 0$$

$$x \geq -\frac{1}{2}$$

$$4. \quad 5^x < 3 \cdot 25^x + 5^{x+1}$$

$$5^x - 3 \cdot 5^{2x} - 5 \cdot 5^x < 0$$

$$-3 \cdot 5^{2x} - 4 \cdot 5^x < 0$$

$$3 \cdot 5^{2x} + 4 \cdot 5^x > 0$$

$$\forall x \in \mathbb{R}$$

$$5. \quad \log_{\sqrt{2}}(x-4) - \log_{\sqrt{2}}(2x-3) = \log_{\sqrt{2}}(2-x)$$

$$\text{c.a.: } \begin{cases} x-4 > 0 \\ 2x-3 > 0 \\ 2-x > 0 \end{cases}$$

$$\begin{cases} x > 4 \\ x > \frac{3}{2} \\ x < 2 \end{cases}$$

imp.

$$6. \quad \log_2 (2x - 1) - \log_2 (x + 3) = 3$$

$$c.a.: \begin{cases} 2x - 1 > 0 \\ x + 3 > 0 \end{cases} \quad \begin{cases} x > \frac{1}{2} \\ x > -3 \end{cases} \quad c.a.: x > \frac{1}{2}$$

$$\log_2 (2x - 1) = \log_2 (x + 3) + \log_2 8 \quad \log_2 (2x - 1) = \log_2 [8(x + 3)]$$

$$2x - 1 = 8(x + 3) \quad 2x - 1 = 8x + 24$$

$$6x = -25 \quad x = -\frac{25}{6} \text{ non accettabile per c.a.} \Rightarrow \text{imp.}$$

$$7. \quad \ln (5 - x) = 2 \ln (x + 1)$$

$$c.a.: \begin{cases} 5 - x > 0 \\ x + 1 > 0 \end{cases} \quad \begin{cases} x < 5 \\ x > -1 \end{cases} \quad c.a.: -1 < x < 5$$

$$\ln (5 - x) = \ln (x + 1)^2 \quad 5 - x = (x + 1)^2$$

$$x^2 + 2x + 1 - 5 + x = 0 \quad x^2 + 3x - 4 = 0$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9 + 16}}{2} \begin{cases} 1 \\ -4 \end{cases} \text{ non acc. per c.a.} \quad x = 1$$

$$8. \quad \log x^3 - 3 = 6 \log x$$

$$c.a.: x > 0 \quad 3 \log x - 3 = 6 \log x$$

$$\log x - 1 = 2 \log x \quad \log x = -1$$

$$x = \frac{1}{10}$$

$$9. \quad \log_{\frac{3}{2}} (x^2 - 1) - \log_{\frac{3}{2}} (x - 2) > 1$$

$$c.a.: \begin{cases} x^2 - 1 > 0 \\ x - 2 > 0 \end{cases} \quad \begin{cases} x < -1 \vee x > 1 \\ x > 2 \end{cases} \quad c.a.: x > 2$$

$$\log_{\frac{3}{2}} (x^2 - 1) > \log_{\frac{3}{2}} (x - 2) + \log_{\frac{3}{2}} \frac{3}{2} \quad \log_{\frac{3}{2}} (x^2 - 1) > \log_{\frac{3}{2}} \left[\frac{3}{2} (x - 2) \right]$$

$$x^2 - 1 > \frac{3}{2} (x - 2) \quad 2x^2 - 2 > 3(x - 2)$$

$$2x^2 - 3x + 4 > 0 \quad x_{1,2} = \frac{3 \pm \sqrt{9 - 32}}{4} \quad \Delta < 0 \Rightarrow \forall x \in R$$

$$\begin{cases} \forall x \in R \\ x > 2 \end{cases} \Rightarrow x > 2$$