1. $2^x + 2^{x-1} + 2^{x-2} = 7$

$2^x + \frac{2^x}{2} + \frac{2^x}{2^2} = 7$ $\qquad$ $2^x \left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7$

$2^x \frac{7}{4} = 7$ $\qquad$ $2^{x-2} = 1$ $\qquad$ $2^{x-2} = 2^0$

$x - 2 = 0$ $\qquad$ $x = 2$

2. $2^{1-x} + 2^{1+x} = 4$

$\frac{2}{2^x} + 2 \cdot 2^x = 4$ $\qquad$ Pongo: $2^x = t$ $\qquad$ $\frac{2}{t} + 2t = 4$

$t^2 - 2t + 1 = 0$ $\qquad$ $(t-1)^2 = 0$

$t = 1 \quad \Rightarrow \quad 2^x = 1 \quad \Rightarrow \quad 2^x = 2^0 \quad \Rightarrow \quad x = 0$

3. $\left(\frac{1}{5}\right)^x - \frac{3}{5} > \frac{2}{5^{1-x}}$

$\frac{1}{5^x} - \frac{3}{5} > \frac{2}{\frac{5}{5^x}}$ $\qquad$ Pongo: $5^x = t$ $\qquad$ $\frac{1}{t} - \frac{3}{5} > \frac{2t}{5}$

$2t^2 + 3t - 5 < 0$ $\qquad$ $t_{1,2} = \frac{-3 \pm \sqrt{9 + 40}}{4} = \frac{-3 \pm 7}{4} = \begin{cases} -\frac{5}{2} \\ 1 \end{cases}$

$-\frac{5}{2} < t < 1 \quad \Rightarrow \quad -\frac{5}{2} < 5^x < 1 \quad \Rightarrow \quad 5^x < 1 \quad \Rightarrow \quad x < 0$

4. $\frac{2^x}{2^x + 1} + \frac{2^x}{2^x + 4} \le 1$

Pongo: $2^x = t$ $\qquad$ $\frac{t}{t+1} + \frac{t}{t+4} \le 1$

$\frac{t\,(t+4) + t\,(t+1) - (t+1)\,(t+4)}{(t+1)\,(t+4)} \le 0$

Posso semplificare il denominatore in quanto sicuramente positivo per qualsiasi valore assunto da x

$t^2 + 4t + t^2 + t - t^2 - 4t - t - 4 \le 0$ $\qquad$ $t^2 - 4 \le 0$

$-2 \le t \le 2 \quad \Rightarrow \quad -2 \le 2^x \le 2 \quad \Rightarrow \quad 2^x \le 2 \quad \Rightarrow \quad x \le 1$

5. $2\log_{2/3} x + \log_{2/3} 3 = \log_{2/3}(5x-2)$

$c.a.:\begin{cases} x>0 \\ 5x-2>0 \end{cases}$ $\begin{cases} x>0 \\ x>\dfrac{2}{5} \end{cases}$ $c.a.: x > \dfrac{2}{5}$

$\log_{2/3} x^2 + \log_{2/3} 3 = \log_{2/3}(5x-2)$ $\log_{2/3} 3x^2 = \log_{2/3}(5x-2)$

$3x^2 = 5x-2$ $3x^2 - 5x + 2 = 0$

$x_{1,2} = \dfrac{5 \pm \sqrt{25-24}}{6} = \left\langle \begin{matrix} 1 \\ \dfrac{2}{3} \end{matrix} \right.$ $x = 1; \quad x = \dfrac{2}{3}$

6. $\dfrac{1}{2}\log(x+20) = \log 2 + \dfrac{1}{4}\log(x+20)$

$c.a.: x > -20$

$\dfrac{1}{4}\log(x+20) = \log 2$ $\log(x+20) = 4\log 2$

$\log(x+20) = \log 16$ $x + 20 = 16$ $x = -4$

7. $\ln(10-2x) = \ln(5-x) - \ln 4$

$c.a.:\begin{cases} 10-2x>0 \\ 5-x>0 \end{cases}$ $\begin{cases} x<5 \\ x<5 \end{cases}$ $c.a.: x < 5$

$\ln(10-2x) + \ln 4 = \ln(5-x)$ $\ln 4(10-2x) = \ln(5-x)$

$8(5-x) = 5-x$ $x = 5$ non accettabile per c.a. $\Rightarrow$ *imp.*

8. $2\log_2 x^3 - \log_4 x^2 + 1 < 0$

$c.a.: x > 0$ $6\log_2 x - 2\dfrac{\log_2 x}{\log_2 4} + 1 < 0$

$6\log_2 x - \log_2 x + 1 < 0$ $5\log_2 x < -1$ $\log_2 x < -\dfrac{1}{5}$

$x < \sqrt[5]{\dfrac{1}{2}}$ Mettendo a sistema con le condizioni di accettabilità, risulta: $0 < x < \sqrt[5]{\dfrac{1}{2}}$

9. $\log_{2/5} \frac{x+1}{x-1} \le 0$

$$\log_{2/5} \frac{x+1}{x-1} \le \log_{2/5} 1$$

$$\begin{cases} \frac{x+1}{x-1} > 0 \\ \frac{x+1}{x-1} \ge 1 \end{cases}$$

Questo sistema equivale alla disequazione: $\frac{x+1}{x-1} \ge 1$

$$\frac{x+1-x+1}{x-1} \ge 0 \qquad \frac{2}{x-1} \ge 0 \qquad x > 1$$